

Class 4: Truth Tables in TFL

PHILOSOPHY 201: INTRODUCTION TO LOGIC
WITH ZEE PERRY

First, some admin

Continue to use this website to access the book & syllabi:

- www.zrperry.com/logic201-spring2020

This site will *also* be populated with:

- PDF versions of the slides from previous classes (for review or as a supplemental source when studying)
- Future homework assignments.

This week's HOMEWORK:

Chapter 10

- Question Block "A", question 2 plus all the **odd** questions
- Question Block "B", questions 2 and 4
- Question Block "C", questions 1, 2, and 3
- Question Block "D", question 4

Due Monday, either *in-class* or emailed *before* class starts!

Last week's HOMEWORK:

We'll do some quick review

Chapter-Questions:

1-2 and 1-4, 2-A and 2-B, 3-C and 3-E,
5-B-(just the even questions) and 5-E

Can be turned in via email to:

Zee.Perry@Rutgers.edu

Chapter 1

2. It must have been sunny. I did wear my sunglasses, after all.

4. Miss Scarlett and Professor Plum were in the study at the time of the murder. Reverend Green had the candlestick in the ballroom, and we know that there is no blood on his hands. Hence Colonel Mustard did it in the kitchen with the lead pipe. Recall, after all, that the gun had not been fired.

Chapter 2-A (the bolded args are valid)

1. Socrates is a man
2. All men are carrots.
- ∴ Socrates is a carrot.

1. Abe Lincoln was either born in Illinois or he was once president.
2. Abe Lincoln was never president.
- ∴ Abe Lincoln was born in Illinois.

1. If I pull the trigger, Abe Lincoln will die.
2. I do not pull the trigger.
- ∴ Abe Lincoln will not die.

1. Abe Lincoln was either from France or from Luxemborg.
2. Abe Lincoln was not from Luxemborg.
- ∴ Abe Lincoln was from France.

1. If the world ends today, then I will not need to get up tomorrow morning.
2. I will need to get up tomorrow morning.
- ∴ The world will not end today.

1. Joe is now 19 years old.
2. Joe is now 87 years old.
- ∴ Bob is now 20 years old.

Chapter 2-B.. Could there be..?

1. A valid argument that has one false premise and one true premise?
2. A valid argument that has only false premises?
3. A valid argument with only false premises and a false conclusion?
4. An invalid argument that can be made valid by the addition of a new premise?
5. A valid argument that can be made invalid by the addition of a new premise?

Chapter 2-B.. Could there be..?

- Y 1. A valid argument that has one false premise and one true premise?
- Y 2. A valid argument that has only false premises?
- Y 3. A valid argument with only false premises and a false conclusion?
- Y 4. An invalid argument that can be made valid by the addition of a new premise?
- N ~~5. A valid argument that can be made invalid by the addition of a new premise?~~

5-E

1. Alice and Bob are both spies.
2. If either Alice or Bob is a spy, then the code has been broken.
3. If neither Alice nor Bob is a spy, then the code remains unbroken.
4. The German embassy will be in an uproar, unless someone has broken the code.
5. Either the code has been broken or it has not, but the German embassy will be in an uproar regardless.
6. Either Alice or Bob is a spy, but not both.

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- A: Alice is a Spy
- B: Bob is a spy
- C: Code has been broken
- D: German Embassy will be in an uproar

- 1.
- 2.
- 3.
- 4.
- 5.
- 6.

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1. $(A \wedge B)$
- 2.
- 3.
- 4.
- 5.
- 6.

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1. $(A \wedge B)$
2. $((A \vee B) \rightarrow C)$
- 3.
- 4.
- 5.
- 6.

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1. $(A \wedge B)$
2. $((A \vee B) \rightarrow C)$
3. $(\neg(A \vee B) \rightarrow \neg C)$
- 4.
- 5.
- 6.

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B: Bob is a spy
C: Code has been broken
D: German Embassy will be in an uproar

1. $(A \wedge B)$
2. $((A \vee B) \rightarrow C)$
3. $(\neg(A \vee B) \rightarrow \neg C)$
4. $(D \vee C)$
- 5.
- 6.

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B: Bob is a spy
C: Code has been broken
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1. $(A \wedge B)$
2. $((A \vee B) \rightarrow C)$
3. $(\neg(A \vee B) \rightarrow \neg C)$
4. $(\neg C \rightarrow D)$
- 5.
- 6.

5-E

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2. $((A \vee B) \rightarrow C)$
3. $(\neg(A \vee B) \rightarrow \neg C)$
4. $(D \vee C)$
- 5.
- 6.

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D: German Embassy will be in an uproar

1. $(A \wedge B)$
2. $((A \vee B) \rightarrow C)$
3. $(\neg(A \vee B) \rightarrow \neg C)$
4. $(D \vee C)$
5. $(C \vee \neg C) \wedge D$
- 6.

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C: Code has been broken
D: German Embassy will be in an uproar

1. $(A \wedge B)$
2. $((A \vee B) \rightarrow C)$
3. $(\neg(A \vee B) \rightarrow \neg C)$
4. $(D \vee C)$
5. $(C \vee \neg C) \wedge D$
6. $((A \vee B) \wedge \neg(A \wedge B))$

Review of Previous Class:

Valid arguments hold in “every case”

If an argument is **Valid**, its premises imply its conclusion no matter what.

- You can dream up any scenario you want, and, so long as the premises are true in the scenario, then the conclusion **must** be true (assuming the argument is valid)

Invalid arguments have Counterexamples.

- Counterexamples are: “cases” that show the argument is bad/invalid.
- Where a “case” is a possible hypothetical scenario (**waves hands**)

Review of Previous Class: Logical Consequence

A sentence, call it "C", is a **logical consequence** of a list of other sentences, call them "P₁", "P₂", "P₃" (and so on until "P_n"), if and only if there is **no case** where P₁, P₂, ..., P_n are all true and C is not true.

An argument is **Valid** if its conclusion is a **logical consequence** of its premises.

C	is a (logical) consequence of	P ₁ , P ₂ , ..., and P _n
C	follows from	P ₁ , P ₂ , ..., and P _n
P ₁ , P ₂ , ... and P _n	entail	C

Truth-Functional Logic (TFL)

	Sentence Component	Representation in English (a popular Natural Language)	Representation in TFL
SENTENCES	Atomic Sentences	"I love to eat pizza" "Electrons are point-particles" (like, could be <i>literally any sentence</i>)	$A, B, C, D, E,$ F, G, H, \dots
	SENTENTIAL CONNECTIVES	Negation	"not..", "it's not the case that...", "it's not true that..."
Conjunction		".. and..", ".. but..", ".. however .."	\wedge (Alternatively, '&', '▪')
Disjunction		"Either... or..", ".. or..",	\vee
Material Conditional		"if.. then..", ".. only if .." (or ".. if ..", but only in reverse!)	\rightarrow (Alternatively, '⊃')
Bi-Conditional		".. if and only if..", ".. iff..", "... just in case..",	\leftrightarrow (Alternatively, '≡')

A

T

F

Atomic Sentence:
"A"

A

Case where
A is True:

T

Case where
A is False:

F

<i>A</i>	<i>B</i>
T	T
F	T
T	F
F	F

Atomic Sentence: "B"	<i>A</i>	<i>B</i>
Cases where <i>B</i> is True:	T	T
	F	T
Cases where <i>B</i> is False:	T	F
	F	F

Atomic Sentence: "B"	<i>A</i>	<i>B</i>
Cases where <i>B</i> is True:	T	T
	F	T
Cases where <i>B</i> is False:	T	F
	F	F

<i>A</i>	<i>B</i>	<i>C</i>
T	T	T
F	T	T
T	F	T
F	F	T
T	T	F
F	T	F
T	F	F
F	F	F

Atomic Sentence: "C"	<i>A</i>	<i>B</i>	<i>C</i>
Cases where C is True:	T	T	T
	F	T	T
	T	F	T
	F	F	T
	T	T	F
Cases where C is False:	F	T	F
	T	F	F
	F	F	F

Atomic Sentence: "C"	<i>A</i>	<i>B</i>	<i>C</i>
Cases where C is True:	T	T	T
	F	T	T
	T	F	T
	F	F	T
Cases where C is False:	T	T	F
	F	T	F
	T	F	F
	F	F	F

Atomic Sentence: "A"	<i>A</i>	<i>B</i>	<i>C</i>
Case where A is True:	T	T	T
Case where A is False:	F	T	T
	T	F	T
	F	F	T
	T	T	F
	F	T	F
	T	F	F
	F	F	F

Atomic Sentence: "A"	<i>A</i>	<i>B</i>	<i>C</i>
Case where A is True:	T	T	T
Case where A is False:	F	T	T
	T	F	T
	F	F	T
	T	T	F
	F	T	F
	T	F	F
	F	F	F

Atomic Sentence: "A"	<i>A</i>	<i>B</i>	<i>C</i>
Case where A is True:	T	T	T
Case where A is False:	F	T	T
Case where A is True:	T	F	T
Case where A is False:	F	F	T
Case where A is True:	T	T	F
Case where A is False:	F	T	F
Case where A is True:	T	F	F
Case where A is False:	F	F	F

Atomic Sentence: "A"	<i>A</i>	<i>B</i>	<i>C</i>
Case where A is True:	T	T	T
Case where A is False:	F	T	T
Case where A is True:	T	F	T
Case where A is False:	F	F	T
Case where A is True:	T	T	F
Case where A is False:	F	T	F
Case where A is True:	T	F	F
Case where A is False:	F	F	F

<i>A</i>	<i>B</i>	<i>C</i>
T	T	T
F	T	T
T	F	T
F	F	T
T	T	F
F	T	F
T	F	F
F	F	F

<i>A</i>	<i>B</i>	<i>C</i>
T	T	T
F	T	T
T	F	T
F	F	T
T	T	F
F	T	F
T	F	F
F	F	F

Atomic Sentence: "B"	<i>A</i>	<i>B</i>	<i>C</i>
Cases where <i>B</i> is True:	T	T	T
	F	T	T
Cases where <i>B</i> is False:	T	F	T
	F	F	T
	T	T	F
	F	T	F
	T	F	F
	F	F	F

Atomic Sentence: "B"	<i>A</i>	<i>B</i>	<i>C</i>
Cases where <i>B</i> is True:	T	T	T
	F	T	T
Cases where <i>B</i> is False:	T	F	T
	F	F	T
	T	T	F
	F	T	F
	T	F	F
	F	F	F

Atomic Sentence: "B"	<i>A</i>	<i>B</i>	<i>C</i>
Cases where <i>B</i> is True:	T	T	T
	F	T	T
Cases where <i>B</i> is False:	T	F	T
	F	F	T
Cases where <i>B</i> is True:	T	T	F
	F	T	F
Cases where <i>B</i> is False:	T	F	F
	F	F	F

Atomic Sentence: "B"	<i>A</i>	<i>B</i>	<i>C</i>
Cases where <i>B</i> is True:	T	T	T
	F	T	T
Cases where <i>B</i> is False:	T	F	T
	F	F	T
Cases where <i>B</i> is True:	T	T	F
	F	T	F
Cases where <i>B</i> is False:	T	F	F
	F	F	F

<i>A</i>	<i>B</i>	<i>C</i>
T	T	T
F	T	T
T	F	T
F	F	T
T	T	F
F	T	F
T	F	F
F	F	F

<i>A</i>	<i>B</i>	<i>C</i>
T	T	T
F	T	T
T	F	T
F	F	T
T	T	F
F	T	F
T	F	F
F	F	F

Atomic Sentence: "C"	<i>A</i>	<i>B</i>	<i>C</i>
Cases where C is True:	T	T	T
	F	T	T
	T	F	T
	F	F	T
	T	T	F
Cases where C is False:	F	T	F
	T	F	F
	F	F	F

Atomic Sentence: "C"	<i>A</i>	<i>B</i>	<i>C</i>
Cases where C is True:	T	T	T
	F	T	T
	T	F	T
	F	F	T
Cases where C is False:	T	T	F
	F	T	F
	T	F	F
	F	F	F

<i>A</i>	<i>B</i>	<i>C</i>
T	T	T
F	T	T
T	F	T
F	F	T
T	T	F
F	T	F
T	F	F
F	F	F

<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>
T	T	T	T
F	T	T	T
T	F	T	T
F	F	T	T
T	T	F	T
F	T	F	T
T	F	F	T
F	F	F	T
T	T	T	F
F	T	T	F
T	F	T	F
F	F	T	F
T	T	F	F
F	T	F	F
T	F	F	F
F	F	F	F

How to make a sentence in TFL:

Rule 1: Any Sentence-Letter is a sentence.

- E.g. "A" "C" "Q" "X" "Y" etc.

Rule 2: For the one-place connective (i.e. " \neg "):

- If " \mathcal{A} " is any sentence, then " $\neg \mathcal{A}$ " is a sentence too.

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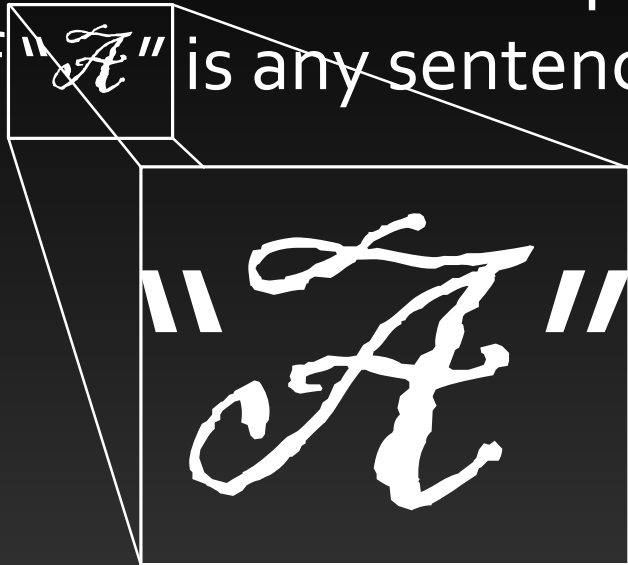
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This is NOT a Sentence-Letter!!!

It's a variable that can stand for any sentence in TFL!!

\mathcal{A} could be "A", " $\neg R$ ", " $(P \wedge Q)$ ", etc.

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" $(\mathcal{A} \vee \mathcal{B})$ " is a sentence,
" $(\mathcal{A} \rightarrow \mathcal{B})$ " is a sentence,
and " $(\mathcal{A} \leftrightarrow \mathcal{B})$ " is a sentence.

Let's make a sentence!

Given the rules we've just discussed, then we can construct *new sentences* in TFL by combining and modifying *other sentences* in TFL.

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Given the rules we've just discussed, then we can construct *new sentences* in TFL by combining and modifying *other sentences* in TFL.

So, given that " P " and " Q " are both sentences (Rule 1), we know that: " $\neg P$ " is a sentence, " $(P \wedge Q)$ " is a sentence, " $(Q \leftrightarrow Q)$ " is a sentence... (Rules 2 and 3).

Quizlet Q1: Let's make a sentence!

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Using these three sentences, construct three **new** sentences in TFL

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Aside: Remember “cases”?

When talking about validity and logical consequence, we talked a lot about “cases”, which were something kinda like “possible, hypothetical scenarios”.

Now that we’re in TFL, the language is simple enough that we can make the concept of a “case” perfectly precise!

- A case (in TFL) is **any** distribution of truth-values to **each** of the ***atomic sentences*** (in whatever argument/sentence you’re considering)

$(A \wedge B)$

A	\wedge	B
T		
F		
T		
F		

$(A \vee B)$

A	\vee	B

$(A \rightarrow B)$

A	\rightarrow	B

$(A \leftrightarrow B)$

A	\leftrightarrow	B

$$(A \wedge B)$$

A	\wedge	B
T		
F		
T		
F		

$$(A \vee B)$$

A	\vee	B
T		
F		
T		
F		

$$(A \rightarrow B)$$

A	\rightarrow	B
T		
F		
T		
F		

$$(A \leftrightarrow B)$$

A	\leftrightarrow	B
T		
F		
T		
F		

$(A \wedge B)$

A	\wedge	B
T		T
F		T
T		F
F		F

$(A \vee B)$

A	\vee	B
T		
F		
T		
F		

$(A \rightarrow B)$

A	\rightarrow	B
T		
F		
T		
F		

$(A \leftrightarrow B)$

A	\leftrightarrow	B
T		
F		
T		
F		

$$(A \wedge B)$$

A	\wedge	B
T		T
F		T
T		F
F		F

$$(A \vee B)$$

A	\vee	B
T		T
F		T
T		F
F		F

$$(A \rightarrow B)$$

A	\rightarrow	B
T		T
F		T
T		F
F		F

$$(A \leftrightarrow B)$$

A	\leftrightarrow	B
T		T
F		T
T		F
F		F

$(A \wedge B)$

A	\wedge	B
T		T
F		T
T		F
F		F

$(A \vee B)$

A	\vee	B
T		T
F		T
T		F
F		F

$(A \rightarrow B)$

A	\rightarrow	B
T		T
F		T
T		F
F		F

$(A \leftrightarrow B)$

A	\leftrightarrow	B
T		T
F		T
T		F
F		F

$$(A \wedge B)$$

A	\wedge	B
T	T	T
F	F	T
T	F	F
F	F	F

$$(A \vee B)$$

A	\vee	B
T		T
F		T
T		F
F		F

$$(A \rightarrow B)$$

A	\rightarrow	B
T		T
F		T
T		F
F		F

$$(A \leftrightarrow B)$$

A	\leftrightarrow	B
T		T
F		T
T		F
F		F

$$(A \wedge B)$$

A	\wedge	B
T	T	T
F	F	T
T	F	F
F	F	F

$$(A \vee B)$$

A	\vee	B
T		T
F		T
T		F
F		F

$$(A \rightarrow B)$$

A	\rightarrow	B
T		T
F		T
T		F
F		F

$$(A \leftrightarrow B)$$

A	\leftrightarrow	B
T		T
F		T
T		F
F		F

$(A \wedge B)$

A	\wedge	B
T	T	T
F	F	T
T	F	F
F	F	F

$(A \vee B)$

A	\vee	B
T		T
F		T
T		F
F		F

$(A \rightarrow B)$

A	\rightarrow	B
T		T
F		T
T		F
F		F

$(A \leftrightarrow B)$

A	\leftrightarrow	B
T		T
F		T
T		F
F		F

$(A \wedge B)$

A	\wedge	B
T	T	T
F	F	T
T	F	F
F	F	F

$(A \vee B)$

A	\vee	B
T	T	T
F	T	T
T	T	F
F	F	F

$(A \rightarrow B)$

A	\rightarrow	B
T		T
F		T
T		F
F		F

$(A \leftrightarrow B)$

A	\leftrightarrow	B
T		T
F		T
T		F
F		F

$$(A \wedge B)$$

A	\wedge	B
T	T	T
F	F	T
T	F	F
F	F	F

$$(A \vee B)$$

A	\vee	B
T	T	T
F	T	T
T	T	F
F	F	F

$$(A \rightarrow B)$$

A	\rightarrow	B
T		T
F		T
T		F
F		F

$$(A \leftrightarrow B)$$

A	\leftrightarrow	B
T		T
F		T
T		F
F		F

$(A \wedge B)$

A	\wedge	B
T	T	T
F	F	T
T	F	F
F	F	F

$(A \vee B)$

A	\vee	B
T	T	T
F	T	T
T	T	F
F	F	F

$(A \rightarrow B)$

A	\rightarrow	B
T		T
F		T
T		F
F		F

$(A \leftrightarrow B)$

A	\leftrightarrow	B
T		T
F		T
T		F
F		F

$(A \wedge B)$

A	\wedge	B
T	T	T
F	F	T
T	F	F
F	F	F

$(A \vee B)$

A	\vee	B
T	T	T
F	T	T
T	T	F
F	F	F

$(A \rightarrow B)$

A	\rightarrow	B
T		T
F		T
T		F
F		F

$(A \leftrightarrow B)$

A	\leftrightarrow	B
T		T
F		T
T		F
F		F

$(A \wedge B)$

A	\wedge	B
T	T	T
F	F	T
T	F	F
F	F	F

$(A \vee B)$

A	\vee	B
T	T	T
F	T	T
T	T	F
F	F	F

$(A \rightarrow B)$

A	\rightarrow	B
T		T
F		T
T		F
F		F

$(A \leftrightarrow B)$

A	\leftrightarrow	B
T		T
F		T
T		F
F		F

$(A \wedge B)$

A	\wedge	B
T	T	T
F	F	T
T	F	F
F	F	F

$(A \vee B)$

A	\vee	B
T	T	T
F	T	T
T	T	F
F	F	F

$(A \rightarrow B)$

A	\rightarrow	B
T		T
F		T
T	F	F
F		F

$(A \leftrightarrow B)$

A	\leftrightarrow	B
T		T
F		T
T		F
F		F

$$(A \wedge B)$$

A	\wedge	B
T	T	T
F	F	T
T	F	F
F	F	F

$$(A \vee B)$$

A	\vee	B
T	T	T
F	T	T
T	T	F
F	F	F

$$(A \rightarrow B)$$

A	\rightarrow	B
T	T	T
F	T	T
T	F	F
F	T	F

$$(A \leftrightarrow B)$$

A	\leftrightarrow	B
T		T
F		T
T		F
F		F

$$(A \wedge B)$$

A	\wedge	B
T	T	T
F	F	T
T	F	F
F	F	F

$$(A \vee B)$$

A	\vee	B
T	T	T
F	T	T
T	T	F
F	F	F

$$(A \rightarrow B)$$

A	\rightarrow	B
T	T	T
F	T	T
T	F	F
F	T	F

$$(A \leftrightarrow B)$$

A	\leftrightarrow	B
T	T	T
F	T	T
T	F	F
F	F	F

$(A \wedge B)$

A	\wedge	B
T	T	T
F	F	T
T	F	F
F	F	F

$(A \vee B)$

A	\vee	B
T	T	T
F	T	T
T	T	F
F	F	F

$(A \rightarrow B)$

A	\rightarrow	B
T	T	T
F	T	T
T	F	F
F	T	F

$(A \leftrightarrow B)$

A	\leftrightarrow	B
T		T
F		T
T		F
F		F

$$(A \wedge B)$$

A	\wedge	B
T	T	T
F	F	T
T	F	F
F	F	F

$$(A \vee B)$$

A	\vee	B
T	T	T
F	T	T
T	T	F
F	F	F

$$(A \rightarrow B)$$

A	\rightarrow	B
T	T	T
F	T	T
T	F	F
F	T	F

$$(A \leftrightarrow B)$$

A	\leftrightarrow	B
T	T	T
F	F	T
T	F	F
F	T	F

$(A \wedge B)$

A	\wedge	B
T	T	T
F	F	T
T	F	F
F	F	F

$(A \vee B)$

A	\vee	B
T	T	T
F	T	T
T	T	F
F	F	F

$(A \rightarrow B)$

A	\rightarrow	B
T	T	T
F	T	T
T	F	F
F	T	F

$(A \leftrightarrow B)$

A	\leftrightarrow	B
T	T	T
F		T
T		F
F	T	F

$(A \wedge B)$

A	\wedge	B
T	T	T
F	F	T
T	F	F
F	F	F

$(A \vee B)$

A	\vee	B
T	T	T
F	T	T
T	T	F
F	F	F

$(A \rightarrow B)$

A	\rightarrow	B
T	T	T
F	T	T
T	F	F
F	T	F

$(A \leftrightarrow B)$

A	\leftrightarrow	B
T	T	T
F	F	T
T	F	F
F	T	F

$$(A \wedge B)$$

A	\wedge	B
T	T	T
F	F	T
T	F	F
F	F	F

$$(A \vee B)$$

A	\vee	B
T	T	T
F	T	T
T	T	F
F	F	F

$$(A \rightarrow B)$$

A	\rightarrow	B
T	T	T
F	T	T
T	F	F
F	T	F

$$(A \leftrightarrow B)$$

A	\leftrightarrow	B
T	T	T
F	F	T
T	F	F
F	T	F

$$(A \wedge B)$$

A	\wedge	B
T	T	T
F	F	T
T	F	F
F	F	F

$$(A \vee B)$$

A	\vee	B
T	T	T
F	T	T
T	T	F
F	F	F

$$(A \rightarrow B)$$

A	\rightarrow	B
T	T	T
F	T	T
T	F	F
F	T	F

$$(A \leftrightarrow B)$$

A	\leftrightarrow	B
T	T	T
F	F	T
T	F	F
F	T	F

What about a more complex sentence?

$(A \vee B)$		
A	\vee	B
T		T
F		T
T		F
F		F

$(A \vee B) \leftrightarrow (A \rightarrow B)$		
$(A \vee B)$	\leftrightarrow	$(A \rightarrow B)$

$(A \rightarrow B)$		
A	\rightarrow	B
T		T
F		T
T		F
F		F

What about a more complex sentence?

$(A \vee B)$		
A	\vee	B
T	T	T
F	T	T
T	T	F
F	F	F

$(A \vee B) \leftrightarrow (A \rightarrow B)$		
$(A \vee B)$	\leftrightarrow	$(A \rightarrow B)$

$(A \rightarrow B)$		
A	\rightarrow	B
T		T
F		T
T		F
F		F

What about a more complex sentence?

$(A \vee B)$		
A	\vee	B
T	T	T
F	T	T
T	T	F
F	F	F

$(A \vee B) \leftrightarrow (A \rightarrow B)$		
$(A \vee B)$	\leftrightarrow	$(A \rightarrow B)$

$(A \rightarrow B)$		
A	\rightarrow	B
T	T	T
F	T	T
T	F	F
F	T	F

What about a more complex sentence?

$(A \vee B)$			$(A \vee B) \leftrightarrow (A \rightarrow B)$			$(A \rightarrow B)$		
A	\vee	B	$(A \vee B)$	\leftrightarrow	$(A \rightarrow B)$	A	\rightarrow	B
T	T	T	T			T	T	T
F	T	T	T			F	T	T
T	T	F	T			T	F	F
F	F	F				F	T	F

What about a more complex sentence?

$(A \vee B)$			$(A \vee B) \leftrightarrow (A \rightarrow B)$			$(A \rightarrow B)$		
A	\vee	B	$(A \vee B)$	\leftrightarrow	$(A \rightarrow B)$	A	\rightarrow	B
T	T	T	T			T	T	T
F	T	T	T			F	T	T
T	T	F	T			T	F	F
F	F	F	F			F	T	F

What about a more complex sentence?

$(A \vee B)$			$(A \vee B) \leftrightarrow (A \rightarrow B)$			$(A \rightarrow B)$		
A	\vee	B	$(A \vee B)$	\leftrightarrow	$(A \rightarrow B)$	A	\rightarrow	B
T	T	T	T		T	T	T	T
F	T	T	T		T	F	T	T
T	T	F	T			T	F	F
F	F	F	F		T	F	T	F

What about a more complex sentence?

$(A \vee B)$		
A	\vee	B
T	T	T
F	T	T
T	T	F
F	F	F

$(A \vee B) \leftrightarrow (A \rightarrow B)$		
$(A \vee B)$	\leftrightarrow	$(A \rightarrow B)$
T		T
T		T
T		F
F		T

$(A \rightarrow B)$		
A	\rightarrow	B
T	T	T
F	T	T
T	F	F
F	T	F

What about a more complex sentence?

$(A \vee B)$		
A	\vee	B
T	T	T
F	T	T
T	T	F
F	F	F

$(A \vee B) \leftrightarrow (A \rightarrow B)$		
$(A \vee B)$	\leftrightarrow	$(A \rightarrow B)$
T		T
T		T
T		F
F		T

$(A \rightarrow B)$		
A	\rightarrow	B
T	T	T
F	T	T
T	F	F
F	T	F

What about a more complex sentence?

$(A \vee B)$		
A	\vee	B
T	T	T
F	T	T
T	T	F
F	F	F

$(A \vee B) \leftrightarrow (A \rightarrow B)$		
$(A \vee B)$	\leftrightarrow	$(A \rightarrow B)$
T		T
T		T
T		F
F		T

$(A \rightarrow B)$		
A	\rightarrow	B
T	T	T
F	T	T
T	F	F
F	T	F

What about a more complex sentence?

$(A \vee B)$		
A	\vee	B
T	T	T
F	T	T
T	T	F
F	F	F

$(A \vee B) \leftrightarrow (A \rightarrow B)$		
$(A \vee B)$	\leftrightarrow	$(A \rightarrow B)$
T	T	T
T	T	T
T		F
F		T

$(A \rightarrow B)$		
A	\rightarrow	B
T	T	T
F	T	T
T	F	F
F	T	F

What about a more complex sentence?

$(A \vee B)$		
A	\vee	B
T	T	T
F	T	T
T	T	F
F	F	F

$(A \vee B) \leftrightarrow (A \rightarrow B)$		
$(A \vee B)$	\leftrightarrow	$(A \rightarrow B)$
T	T	T
T	T	T
T		F
F		T

$(A \rightarrow B)$		
A	\rightarrow	B
T	T	T
F	T	T
T	F	F
F	T	F

What about a more complex sentence?

$(A \vee B)$		
A	\vee	B
T	T	T
F	T	T
T	T	F
F	F	F

$(A \vee B) \leftrightarrow (A \rightarrow B)$		
$(A \vee B)$	\leftrightarrow	$(A \rightarrow B)$
T	T	T
T	T	T
T		F
F		T

$(A \rightarrow B)$		
A	\rightarrow	B
T	T	T
F	T	T
T	F	F
F	T	F

What about a more complex sentence?

$(A \vee B)$		
A	\vee	B
T	T	T
F	T	T
T	T	F
F	F	F

$(A \vee B) \leftrightarrow (A \rightarrow B)$		
$(A \vee B)$	\leftrightarrow	$(A \rightarrow B)$
T	T	T
T	T	T
T		F
F		T

$(A \rightarrow B)$		
A	\rightarrow	B
T	T	T
F	T	T
T	F	F
F	T	F

What about a more complex sentence?

$(A \vee B)$		
A	\vee	B
T	T	T
F	T	T
T	T	F
F	F	F

$(A \vee B) \leftrightarrow (A \rightarrow B)$		
$(A \vee B)$	\leftrightarrow	$(A \rightarrow B)$
T	T	T
T	T	T
T	F	F
F	T	T

$(A \rightarrow B)$		
A	\rightarrow	B
T	T	T
F	T	T
T	F	F
F	T	F

What about a more complex sentence?

$(A \vee B)$		
A	\vee	B
T	T	T
F	T	T
T	T	F
F	F	F

$(A \vee B) \leftrightarrow (A \rightarrow B)$		
$(A \vee B)$	\leftrightarrow	$(A \rightarrow B)$
T	T	T
T	T	T
T	F	F
F		T

$(A \rightarrow B)$		
A	\rightarrow	B
T	T	T
F	T	T
T	F	F
F	T	F

What about a more complex sentence?

$(A \vee B)$		
A	\vee	B
T	T	T
F	T	T
T	T	F
F	F	F

$(A \vee B) \leftrightarrow (A \rightarrow B)$		
$(A \vee B)$	\leftrightarrow	$(A \rightarrow B)$
T	T	T
T	T	T
T	F	F
F		T

$(A \rightarrow B)$		
A	\rightarrow	B
T	T	T
F	T	T
T	F	F
F	T	F

What about a more complex sentence?

$(A \vee B)$		
A	\vee	B
T	T	T
F	T	T
T	T	F
F	F	F

$(A \vee B) \leftrightarrow (A \rightarrow B)$		
$(A \vee B)$	\leftrightarrow	$(A \rightarrow B)$
T	T	T
T	T	T
T	F	F
F	F	T

$(A \rightarrow B)$		
A	\rightarrow	B
T	T	T
F	T	T
T	F	F
F	T	F

What about a more complex sentence?

$(A \vee B)$		
A	\vee	B
T	T	T
F	T	T
T	T	F
F	F	F

$(A \vee B) \leftrightarrow (A \rightarrow B)$		
$(A \vee B)$	\leftrightarrow	$(A \rightarrow B)$
T	T	T
T	T	T
T	F	F
F	F	T

$(A \rightarrow B)$		
A	\rightarrow	B
T	T	T
F	T	T
T	F	F
F	T	F

What about a more complex sentence?

$(A \vee B)$			$(A \vee B) \leftrightarrow (A \rightarrow B)$			$(A \rightarrow B)$		
A	\vee	B	$(A \vee B)$	\leftrightarrow	$(A \rightarrow B)$	A	\rightarrow	B
T	T	T	T	T	T	T	T	T
F	T	T	T	T	T	F	T	T
T	T	F	T	F	F	T	F	F
F	F	F	F	F	T	F	T	F

What about a more complex sentence?

$(A \vee B)$		
A	\vee	B
T	T	T
F	T	T
T	T	F
F	F	F

$(A \vee B) \leftrightarrow (A \rightarrow B)$		
$(A \vee B)$	\leftrightarrow	$(A \rightarrow B)$
T	T	T
T	T	T
T	F	F
F	F	T

$(A \rightarrow B)$		
A	\rightarrow	B
T	T	T
F	T	T
T	F	F
F	T	F

What about a more complex sentence?

$\neg A$		$\neg B$	
\neg	A	\neg	B
F	T	F	T
T	F	F	T
F	T	T	F
T	F	T	F

What about a more complex sentence?

$\neg A$		$\neg B$	
\neg	A	\neg	B
F	T	F	T
T	F	F	T
F	T	T	F
T	F	T	F

What about a more complex sentence?

$\neg A$		$\neg B$	
\neg	A	\neg	B
F	T	F	T
T	F	F	T
F	T	T	F
T	F	T	F

What about a more complex sentence?

$\neg A$		$\neg B$	
\neg	A	\neg	B
F	T	F	T
T	F	F	T
F	T	T	F
T	F	T	F

What about a more complex sentence?

$\neg A$		$\neg B$	
\neg	A	\neg	B
F	T	F	T
T	F	F	T
F	T	T	F
T	F	T	F

What about a more complex sentence?

$\neg A$		$\neg B$	
\neg	A	\neg	B
F	T	F	T
T	F	F	T
F	T	T	F
T	F	T	F

What about a more complex sentence?

$\neg A$		$\neg B$		$(\neg B \rightarrow \neg A)$		
\neg	A	\neg	B	$\neg B$	\rightarrow	$\neg A$
F	T	F	T			
T	F	F	T			
F	T	T	F			
T	F	T	F			

What about a more complex sentence?

$\neg A$		$\neg B$		$(\neg B \rightarrow \neg A)$		
\neg	A	\neg	B	$\neg B$	\rightarrow	$\neg A$
F	T	F	T			
T	F	F	T			
F	T	T	F			
T	F	T	F			

What about a more complex sentence?

$\neg A$		$\neg B$		$(\neg B \rightarrow \neg A)$		
\neg	A	\neg	B	$\neg B$	\rightarrow	$\neg A$
F	T	F	T	F		
T	F	F	T	F		
F	T	T	F	T		
T	F	T	F	T		

What about a more complex sentence?

$\neg A$		$\neg B$		$(\neg B \rightarrow \neg A)$		
\neg	A	\neg	B	$\neg B$	\rightarrow	$\neg A$
F	T	F	T	F		
T	F	F	T	F		
F	T	T	F	T		
T	F	T	F	T		

What about a more complex sentence?

$\neg A$		$\neg B$		$(\neg B \rightarrow \neg A)$		
\neg	A	\neg	B	$\neg B$	\rightarrow	$\neg A$
F	T	F	T	F		
T	F	F	T	F		
F	T	T	F	T		
T	F	T	F	T		

What about a more complex sentence?

$\neg A$		$\neg B$		$(\neg B \rightarrow \neg A)$		
\neg	A	\neg	B	$\neg B$	\rightarrow	$\neg A$
F	T	F	T	F		F
T	F	F	T	F		T
F	T	T	F	T		F
T	F	T	F	T		T

What about a more complex sentence?

$\neg A$		$\neg B$		$(\neg B \rightarrow \neg A)$		
\neg	A	\neg	B	$\neg B$	\rightarrow	$\neg A$
F	T	F	T	F		F
T	F	F	T	F		T
F	T	T	F	T		F
T	F	T	F	T		T

What about a more complex sentence?

$\neg A$		$\neg B$		$(\neg B \rightarrow \neg A)$		
\neg	A	\neg	B	$\neg B$	\rightarrow	$\neg A$
F	T	F	T	F		F
T	F	F	T	F		T
F	T	T	F	T		F
T	F	T	F	T		T

What about a more complex sentence?

$\neg A$		$\neg B$		$(\neg B \rightarrow \neg A)$		
\neg	A	\neg	B	$\neg B$	\rightarrow	$\neg A$
F	T	F	T	F		F
T	F	F	T	F		T
F	T	T	F	T	F	F
T	F	T	F	T		T

What about a more complex sentence?

$\neg A$		$\neg B$		$(\neg B \rightarrow \neg A)$		
\neg	A	\neg	B	$\neg B$	\rightarrow	$\neg A$
F	T	F	T	F		F
T	F	F	T	F		T
F	T	T	F	T	F	F
T	F	T	F	T		T

What about a more complex sentence?

$\neg A$		$\neg B$		$(\neg B \rightarrow \neg A)$		
\neg	A	\neg	B	$\neg B$	\rightarrow	$\neg A$
F	T	F	T	F		F
T	F	F	T	F		T
F	T	T	F	T	F	F
T	F	T	F	T		T

What about a more complex sentence?

$\neg A$		$\neg B$		$(\neg B \rightarrow \neg A)$		
\neg	A	\neg	B	$\neg B$	\rightarrow	$\neg A$
F	T	F	T	F		F
T	F	F	T	F		T
F	T	T	F	T	F	F
T	F	T	F	T		T

What about a more complex sentence?

$\neg A$		$\neg B$		$(\neg B \rightarrow \neg A)$		
\neg	A	\neg	B	$\neg B$	\rightarrow	$\neg A$
F	T	F	T	F	T	F
T	F	F	T	F	T	T
F	T	T	F	T	F	F
T	F	T	F	T	T	T

What about a more complex sentence?

$\neg A$		$\neg B$		$(\neg B \rightarrow \neg A)$		
\neg	A	\neg	B	$\neg B$	\rightarrow	$\neg A$
F	T	F	T	F	T	F
T	F	F	T	F	T	T
F	T	T	F	T	F	F
T	F	T	F	T	T	T

What about a more complex sentence?

$\neg A$		$\neg B$		$(\neg B \rightarrow \neg A)$		
\neg	A	\neg	B	$\neg B$	\rightarrow	$\neg A$
F	T	F	T	F	T	F
T	F	F	T	F	T	T
F	T	T	F	T	F	F
T	F	T	F	T	T	T

What about a more complex sentence?

$\neg A$	
\neg	A
F	T
T	F
F	T
T	F

$\neg B$	
\neg	B
F	T
F	T
T	F
T	F

$(\neg B \rightarrow \neg A)$		
$\neg B$	\rightarrow	$\neg A$
F	T	F
F	T	T
T	F	F
T	T	T

$(A \rightarrow B)$		
A	\rightarrow	B
T	T	T
F	T	T
T	F	F
F	T	F

What about a more complex sentence?

$\neg A$	
\neg	A
F	T
T	F
F	T
T	F

$\neg B$	
\neg	B
F	T
F	T
T	F
T	F

$(\neg B \rightarrow \neg A)$		
$\neg B$	\rightarrow	$\neg A$
F	T	F
F	T	T
T	F	F
T	T	T

$(A \rightarrow B)$		
A	\rightarrow	B
T	T	T
F	T	T
T	F	F
F	T	F