# Class 5: Semantics in TFL

PHILOSOPHY 201: INTRODUCTION TO LOGIC

WITH ZEE PERRY

# First, some admin

Continue to use this website to access the book & syllabi:

- www.zrperry.com/logic201-spring2020

This site will *also* be populated with:

- PDF versions of the slides from previous classes (for review or as a supplemental source when studying)
- Future homework assignments.

# This week's HOMEWORK:

## Chapter 10

- Question Block "A", question 2 plus all the **odd** questions
- Question Block "B", questions 2 and 4
- Question Block "C", questions 1, 2, and 3
- Question Block "D", question 4

Due TODAY, either ***in-class*** or emailed ***immediately***!

# By the way:

Next class is our first MIDTERM exam (quiz)

- It will test all the material we've covered so far
- It will consist of questions similar to the assigned homework questions.

- You will be able to bring ONE single-paper* "cheat sheet" that you can write whatever you want on.
  - Otherwise it is closed book/notes.

Block "A", question 2 plus all the odd questions!

2

| $(C \to \neg C)$ | | | |
|---|---|---|---|
| $C$ | $\to$ | $\neg$ | $C$ |
| | | | |
| | | | |

3

| $((A \leftrightarrow B) \leftrightarrow \neg(A \leftrightarrow \neg B))$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $(A$ | $\leftrightarrow$ | $B)$ | $\leftrightarrow$ | $\neg$ | $(A$ | $\leftrightarrow$ | $\neg$ | $B)$ |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |

# Block "A", question 2 plus all the odd questions!

2

$(C \rightarrow \neg C)$

| $C$ | $\rightarrow$ | $\neg$ | $C$ |
|---|---|---|---|
| T | F | F | T |
| F | T | T | F |

3

$((A \leftrightarrow B) \leftrightarrow \neg(A \leftrightarrow \neg B))$

| $(A$ | $\leftrightarrow$ | $B)$ | $\leftrightarrow$ | $\neg$ | $(A$ | $\leftrightarrow$ | $\neg$ | $B)$ |
|---|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | F | F | T |
| F | F | T | T | F | F | T | F | T |
| T | F | F | T | F | T | T | T | F |
| F | T | F | T | T | F | F | T | F |

# Block "A", question 2 plus all the odd questions!

5

$$((A \land B) \to (A \lor B))$$

| $(A$ | $\land$ | $B)$ | $\to$ | $(B$ | $\lor$ | $A)$ |
|---|---|---|---|---|---|---|
| | | | | | | |
| | | | | | | |
| | | | | | | |
| | | | | | | |

# Block "A", question 2 plus all the odd questions!

5

$$((A \wedge B) \to (A \vee B))$$

| $(A$ | $\wedge$ | $B)$ | $\to$ | $(B$ | $\vee$ | $A)$ |
|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T |
| F | F | T | T | T | T | F |
| T | F | F | T | F | T | T |
| F | F | F | T | F | F | F |

Block "A" question 2 plus all the odd questions!

7

$((A \land B) \land \neg (A \land B)) \land C$

| $[(A$ | $\land$ | $B)$ | $\land$ | $\neg$ | $(A$ | $\land$ | $B)]$ | $\land$ | $C$ |
|---|---|---|---|---|---|---|---|---|---|
| T | T | T | F | F | T | T | T | F | T |
| F | F | T | F | T | F | F | T | F | T |
| T | F | F | F | T | T | F | F | F | T |
| F | F | F | F | T | F | F | F | F | T |
| T | T | T | F | F | T | T | T | F | F |
| F | F | T | F | T | F | F | T | F | F |
| T | F | F | F | T | T | F | F | F | F |
| F | F | F | F | T | F | F | F | F | F |

$$\neg[(C \vee A) \vee B]$$

| $\neg$ | $[(C$ | $\vee$ | $A)$ | $\vee$ | $B]$ |
|---|---|---|---|---|---|
| F | T | T | T | T | T |
| F | T | T | F | T | T |
| F | T | T | T | T | F |
| F | T | T | F | T | F |
| F | F | T | T | T | T |
| F | F | F | F | T | T |
| F | F | T | T | T | F |
| T | F | F | F | F | F |

Block "A"
question 2
plus all the
odd questions!

# This week's HOMEWORK:

- Question Block "B", questions 2 and 4

# This week’s HOMEWORK:

- Question Block “C”, questions 1, 2, and 3

# Block “C”, Question Block “C”, questions 1, 2, and 3

1

$\neg((S \leftrightarrow (P \rightarrow S))$

| $\neg$ | $((S$ | $\leftrightarrow$ | $(P$ | $\rightarrow$ | $S))$ |
|---|---|---|---|---|---|
| F | T | T | T | T | T |
| F | F | T | T | F | F |
| F | T | T | F | T | T |
| T | F | F | F | T | F |

2

$\neg((X \wedge Y) \vee (X \vee Y))$

| $\neg$ | $((X$ | $\wedge$ | $Y)$ | $\vee$ | $(X$ | $\vee$ | $Y))$ |
|---|---|---|---|---|---|---|---|
| F | T | T | T | T | T | T | T |
| F | F | F | T | T | F | T | T |
| F | T | F | F | T | T | T | F |
| T | F | F | F | F | F | F | F |

# Block “C”, Question Block “C”, question 3

| $\neg((A \rightarrow B) \leftrightarrow (\neg B \leftrightarrow \neg A))$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $\neg$ | $((A$ | $\rightarrow$ | $B)$ | $\leftrightarrow$ | $(\neg$ | $B$ | $\leftrightarrow$ | $\neg$ | $A))$ |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |

# Block “C”, Question Block “C”, question 3

| $\neg((A \rightarrow B) \leftrightarrow (\neg B \leftrightarrow \neg A))$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $\neg$ | $((A$ | $\rightarrow$ | $B)$ | $\leftrightarrow$ | $(\neg$ | $B$ | $\leftrightarrow$ | $\neg$ | $A))$ |
| F | T | T | T | T | F | T | T | F | T |
| T | F | T | T | F | F | T | F | T | F |
| F | T | F | F | T | T | F | F | F | T |
| F | F | T | F | T | T | F | T | T | F |

# This week’s HOMEWORK:

Question Block “D”, question 4

$$((D \wedge R) \rightarrow I) \rightarrow \neg(D \vee R)$$

Block "D"

Question 4

| $((D$ | $\wedge$ | $R)$ | $\rightarrow$ | $I)$ | $\rightarrow$ | $\neg$ | $(D$ | $\vee$ | $R)$ |
|---|---|---|---|---|---|---|---|---|---|
| T | T | T | T | T | F | F | T | T | T |
| F | F | T | T | T | F | F | F | T | T |
| T | F | F | T | T | F | F | T | T | F |
| F | F | F | T | T | T | T | F | F | F |
| T | T | T | F | F | T | F | T | T | T |
| F | F | T | T | F | F | F | F | T | T |
| T | F | F | F | F | T | F | T | T | F |
| F | F | F | T | F | T | T | F | F | F |

# Semantic Notion: Tautologies

Recall: in TFL, a "possible case" is just the assignment of Truth-Values to each of the ***atomic*** sentences.

- We'll use the book's term, "**Valuation**" or "**Possible Valuation**"

Sentence "$\mathcal{A}$" is a **Tautology** iff it is true in every case

Sentence "$\mathcal{A}$" is a **Tautology** iff it is true on every valuation

The sentence on the right is one of the simplest tautologies.

| $(A \vee \neg A)$ | | | |
|---|---|---|---|
| $A$ | $\vee$ | $\neg$ | $A$ |
| **T** | | | |
| **F** | | | |

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$$(A \vee \neg A)$$

| $A$ | $\vee$ | $\neg$ | $A$ |
|---|---|---|---|
| T | T | F | T |
| F | T | T | F |

# Semantic Notion: Contradictions

Contradictions are the opposite of a Tautology. Rather than always being true, contradictions are always false.

"$\mathcal{A}$" is a **Contradiction** iff it is false <u>in every case</u>

"$\mathcal{A}$" is a **Contradiction** iff it is false <u>on every valuation</u>

The sentence on the right is one of the simplest contradictions

It's very similar to the simple tautology, you just switch out the 'or' ('V') for the 'and' ('∧').

| $(A \wedge \neg A)$ | | | |
|---|---|---|---|
| $A$ | $\wedge$ | $\neg$ | $A$ |
| T | | F | T |
| F | | T | F |

# Semantic Notion: Contradictions

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| $(A \wedge \neg A)$ | | | |
|---|---|---|---|
| $A$ | $\wedge$ | $\neg$ | $A$ |
| T | F | F | T |
| F | F | T | F |

# Semantic Notion: (TF)-Logically Equivalent

You'll recall that we had some homework questions about logical equivalence in a ***natural*** language. With the tools of TFL and truth-tables, we can now understand logical equivalence in our formal language as well.

"$\mathcal{A}$" and "$\mathcal{B}$" are **Equivalent** iff they have the *same* truth-value on every valuation.

"$\mathcal{A}$" and "$\mathcal{B}$" are **Equivalent** iff the sentence "$(\mathcal{A} \leftrightarrow \mathcal{B})$" is a ***tautology***.

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"$\mathcal{A}$" and "$\mathcal{B}$" are **Equivalent** iff the sentence "$(\mathcal{A} \leftrightarrow \mathcal{B})$" is a ***tautology***.

| $((\neg A \wedge \neg B) \leftrightarrow \neg (A \vee B))$ | | | | | | | |
|---|---|---|---|---|---|---|---|
| $A$ | $B$ | $(\neg A$ | $\wedge$ | $\neg B)$ | $\leftrightarrow$ | $\neg$ | $(A \vee B)$ |
| T | T | | | | | | |
| F | T | | | | | | |
| T | F | | | | | | |
| F | F | | | | | | |

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| $((\neg A \wedge \neg B) \leftrightarrow \neg (A \vee B))$ | | | | | | | |
|---|---|---|---|---|---|---|---|
| $A$ | $B$ | $(\neg A$ | $\wedge$ | $\neg B)$ | $\leftrightarrow$ | $\neg$ | $(A \vee B)$ |
| T | T | F | F | F | T | T | T |
| F | T | T | F | F | T | T | T |
| T | F | F | F | T | T | F | T |
| F | F | T | T | T | T | F | F |

# Semantic Notion: Jointly Satisfiable

We also had homework questions about when pairs of sentences in ***natural*** language were "Jointly possible" or "jointly impossible". With the tools of TFL and truth-tables, we can now understand a very similar semantic notion: **joint satisfiability**.

"$\mathscr{A}_1$" "$\mathscr{A}_2$",…, "$\mathscr{A}_n$" are **Jointly Satisfiable** iff there exists *some* valuation which makes them all true.

| $(A \wedge (A \vee B))$ | | | | |
|---|---|---|---|---|
| $A$ | $\wedge$ | $(A$ | $\vee$ | $B)$ |
| T | T | T | T | T |
| F | F | F | T | T |
| T | T | T | T | F |
| F | F | F | F | F |

# Semantic Notion: Jointly Satisfiable

We also had homework questions about when pairs of sentences in ***natural*** language were "Jointly possible" or "jointly impossible". With the tools of TFL and truth-tables, we can now understand a very similar semantic notion: **joint satisfiability**.

"$\mathcal{A}_1$" "$\mathcal{A}_2$",…, "$\mathcal{A}_n$" are **Jointly Satisfiable** iff
there exists *some* valuation which makes them all true.

"$\mathcal{A}_1$" "$\mathcal{A}_2$",…, "$\mathcal{A}_n$" are **Jointly Unsatisfiable** iff
there exists *some* valuation which makes them all true.

| $(A \wedge (A \vee B))$ | | | | |
|---|---|---|---|---|
| $A$ | $\wedge$ | $(A$ | $\vee$ | $B)$ |
| T | T | T | T | T |
| F | F | F | T | T |
| T | T | T | T | F |
| F | F | F | F | F |

# Finally, we can get *precise* about: Entailment and Validity

The sentences, "$\mathscr{A}_1$" "$\mathscr{A}_2$",…, "$\mathscr{A}_n$" **Entail** (in TFL) the sentence "$\mathscr{B}$" if there is no single valuation that makes all of "$\mathscr{A}_1$" "$\mathscr{A}_2$",…, "$\mathscr{A}_n$" true and makes "$\mathscr{B}$" false.

If "$\mathscr{A}_1$" "$\mathscr{A}_2$",…, "$\mathscr{A}_n$" **entail** "$\mathscr{B}$", then the argument:

1. $\mathscr{A}_1$
2. $\mathscr{A}_2$

…..

n. $\mathscr{A}_n$

---

c. $\mathscr{B}$

is **VALID!**

(premises) (conclusion)

| $\neg P$ | |
|---|---|
| $\neg$ | $P$ |
| F | T |
| T | F |
| F | T |
| T | F |

| $(P \vee Q)$ | | |
|---|---|---|
| $P$ | $\vee$ | $Q$ |
| T | T | T |
| F | T | T |
| T | T | F |
| F | F | F |

| $Q$ |
|---|
| $Q$ |
| T |
| T |
| F |
| F |

(premises)

(conclusion)

| $\neg P$ | |
|---|---|
| $\neg$ | $P$ |
| F | T |
| T | F |
| F | T |
| T | F |

| $(P \to Q)$ | | |
|---|---|---|
| $P$ | $\to$ | $Q$ |
| T | T | T |
| F | T | T |
| T | F | F |
| F | T | F |

| $Q$ |
|---|
| $Q$ |
| T |
| T |
| F |
| F |

# Let's introduce a New SYMBOL for an OLD language (specifically, English)

## Double turnstile: '⊨'

- So-called because it looks like a turnstile you'd see on a subway platform, an amusement park, a stadium, etc.
- Except that it's "double" because it's got *two* little bars sticking out the side instead of one.

## The double turnstile is ***NOT*** a symbol in TFL!!

It's a symbol of English! Or, a symbol that we're *adding to* English to make it easier for **us** to **talk about logic**.

| $(A \wedge \neg A)$ | | | |
|---|---|---|---|
| $A$ | $\wedge$ | $\neg$ | $A$ |
| T | F | F | T |
| F | F | T | F |

# Let's introduce a New SYMBOL for an OLD language (specifically, English)

## Double turnstile: '$\vDash$'

- So-called because it looks like a turnstile you'd see on a subway platform, an amusement park, a stadium, etc.
- Except that it's "double" because it's got *two* little bars sticking out the side instead of one.

The double-turnstile is a simple way for us to write down statements about **entailment**:

So:

" $P \vee C, \neg P, Q \vDash C$ "

Stands for:

"The sentences '$P \vee C$' '$\neg P$', and '$Q$' **entail** '$C$'. "

which, as we just discussed, also means:

"The argument with premises: '$P \vee C$' '$\neg P$', and '$Q$', and conclusion '$C$' is **valid**. "

| $(A \wedge \neg A)$ | | | |
|---|---|---|---|
| $A$ | $\wedge$ | $\neg$ | $A$ |
| T | F | F | T |
| F | F | T | F |