

Logic Homework 3

Introduction to Logic - Zee Perry

Due February 26th, **IN CLASS!**
(reminder: No class on Monday, Feb 24th)

Directions: The following questions will make use of the Natural Deduction Proof System introduced in Chapter 15. The proofs we'll consider will make use of the following rules (some of which were discussed in class, and the others which are explained in detail in Chapter 15): \wedge INTRODUCTION, \wedge ELIMINATION, REITERATION, \vee INTRODUCTION, \rightarrow ELIMINATION, and \leftrightarrow ELIMINATION. The other rules introduced in that chapter will be covered when we next have class.

1 Correct some incorrect proofs:

The following "proofs" are *incorrect!* Circle the **mistakes** (e.g. mis-uses of rules, formatting mistakes, etc.) that they make and explain what they did wrong:

	1		$(\neg B \wedge C) \rightarrow D$	Pr.
	2		$(C \wedge \neg A) \leftrightarrow (\neg A \wedge \neg B)$	
	3		$\neg A \wedge C$	Pr.
	4		C	$\wedge E, 3$
	5		$\neg A$	$\wedge E, 1$
	6		$C \wedge \neg A$	$\wedge E, 4, 5$
First	7		$\neg A \wedge \neg B$	$\leftrightarrow E, 2, 6$
Incorrect	8		$\neg B$	$\wedge E, 7$
"Proof":	9		$\neg B \wedge C$	$\wedge I, 4, 8$
	10		D	Reit., 11
	11		D	$\rightarrow E, 9$
	12		$D \wedge D$	$\wedge I, 10$
Want to Show:			$(D \wedge D) \vee E$	(Oh, forgot to mention this's the conclusion I want)
	13		$(D \wedge D) \vee E$	$\vee I, 12$

	1	$(P \vee Q) \wedge S$	Pr.
	2	$P \wedge \neg R$	Pr.
Want to Show:		$(\neg R \wedge S) \wedge (S \wedge \neg R)$	Concl.
	3	S	Reit., 1
Second Incorrect "Proof":	4	$P \vee Q$	$\wedge E$, 1
	5	$P \wedge \neg R$	Reit., 2
	6	$\neg R$	$\wedge E$, 5
	7	$\neg R \wedge S$	$\wedge I$, 3, 6
	8	$S \wedge \neg R$	$\wedge I$, 7
	9	$(\neg R \wedge S) \wedge (S \wedge \neg R)$	$\wedge I$, 3, 6, 7, 8
	10	$(\neg R \wedge S) \wedge (S \wedge \neg R)$	Reit., 9

2 Fill in some incomplete proofs!

The following two proofs are incomplete, one is lacking the **rules** and **line numbers** in the right-hand margin. The other is lacking the **TFL Sentences** on each line below the premises. Using the other information contained within each argument, fill in the blanks with the rules, numbers, and sentences (as appropriate) that would produce a legitimate natural deduction proof.

Proof without Rules & Line Numbers

	1	$P \wedge S$	_____ , _____
	2	$S \rightarrow R$	_____ , _____
Want to Show:		$R \vee E$	
	3	P	_____ , _____
	4	S	_____ , _____
	5	R	_____ , _____
	6	$R \vee E$	_____ , _____

Proof without TFL Sentences after Premises.

	1	$(Q \wedge P) \rightarrow S$	Pr.
	2	$P \wedge Q$	Pr.
Want to Show:		$S \wedge Q$	
	3	_____	$\wedge E$, 2
	4	_____	$\wedge E$, 2
	5	_____	$\wedge I$, 3, 4
	6	_____	$\rightarrow E$, 1, 5
	7	_____	$\wedge I$, 4, 6

3 Build Some Proofs!

Pick two of the following arguments, and write out proofs for them using our natural deduction proof system:

$$1. (A \vee C) \wedge D$$

$$2. (A \vee C) \rightarrow B$$

$$\therefore B \wedge D$$

$$1. X \wedge Y$$

$$\therefore (Y \wedge X) \wedge (X \wedge X)$$

$$1. X \wedge Y$$

$$\therefore (Y \wedge X) \vee Z$$

$$1. (A \vee C) \wedge D$$

$$2. (A \vee C) \rightarrow B$$

$$\therefore B \wedge D$$

$$1. (F \vee G) \leftrightarrow H$$

$$2. (\neg G \wedge H) \wedge (\neg K \vee H)$$

$$3. (F \vee G) \rightarrow K$$

$$\therefore K$$