

# Answer Key for Unit 2 Homeworks

Introduction to Logic - Zee Perry

Don't forget! Midterm exam on the 11th!

## 1 Logic Homework 3

### 1.1 Correct some incorrect proofs:

The following “proofs” are *incorrect*! Circle the **mistakes** (e.g. mis-uses of rules, formatting mistakes, etc.) that they make and explain what they did wrong:

First  
Incorrect  
“Proof”:

1	$(\neg B \wedge C) \rightarrow D$	Pr.
2	$(C \wedge \neg A) \leftrightarrow (\neg A \wedge \neg B)$	Pr.
3	$\neg A \wedge C$	Pr.
4	$C$	$\wedge E, 3$
5	$\neg A$	$\wedge E, \cancel{3}$
6	$C \wedge \neg A$	$\wedge I, 4, 5$
7	$\neg A \wedge \neg B$	$\leftrightarrow E, 2, 6$
8	$\neg B$	$\wedge E, 7$
9	$\neg B \wedge C$	$\wedge I, 4, 8$
10	$D$	<del>Reit., 11</del> $\rightarrow E, 9$
11	$D$	$\nrightarrow E, 9$ <del>Reit., 10</del>
12	$D \wedge D$	$\wedge I, 10, 11$
WTS:	$(D \wedge D) \vee E$	(Oh, forgot to mention this's the conclusion I w
13	$(D \wedge D) \vee E$	$\vee I, 12$

Second  
Incorrect  
"Proof":

1		$(P \vee Q) \wedge S$	Pr.
2		$P \wedge \neg R$	Pr.
		<b>WTS: /</b> $(\neg R \wedge S) \wedge (S \wedge \neg R)$	<del>Concl.</del>
3		$S$	<del>Reit.</del> $\wedge E, 1$
4		$P \vee Q$	$\wedge E, 1$
5		$P \wedge \neg R$	Reit., 2
6		$\neg R$	$\wedge E, 5$
7		$\neg R \wedge S$	$\wedge I, 3, 6$
8		$S \wedge \neg R$	$\wedge I, 7, 3, 6$
9		$(\neg R \wedge S) \wedge (S \wedge \neg R)$	$\wedge I, 3, 6, 7, 8$
10		$(\neg R \wedge S) \wedge (S \wedge \neg R)$	Reit., 9

## 1.2 Fill in some incomplete proofs!

### Proof without Rules & Line Numbers

1		$P \wedge S$	Pr.
2		$S \rightarrow R$	Pr.
		<b>WTS:</b> $R \vee E$	
3		$P$	$\wedge E, 1$
4		$S$	$\wedge E, 1$
5		$R$	$\rightarrow E, 2, 4$
6		$R \vee E$	$\vee I, 5$

### Proof without TFL Sentences after Premises.

1		$(Q \wedge P) \rightarrow S$	Pr.
2		$P \wedge Q$	Pr.
		<b>WTS:</b> $S \wedge Q$	
3		$P$	$\wedge E, 2$
4		$Q$	$\wedge E, 2$
5		$Q \wedge P$	$\wedge I, 3, 4$
6		$S$	$\rightarrow E, 1, 5$
7		$S \wedge Q$	$\wedge I, 4, 6$

### 1.3 Build Some Proofs!

$$1. (A \vee C) \wedge D$$

$$2. (A \vee C) \rightarrow B$$

$$\therefore B \wedge D$$

1	$(A \vee C)D$	Pr.
2	$(A \vee C) \rightarrow B$	Pr.
<b>WTS:</b> $B \wedge D$		
3	$D$	$\wedge E, 1$
4	$A \vee C$	$\wedge E, 1$
5	$B$	$\rightarrow E, 2, 4$
6	$B \wedge D$	$\wedge I, 3, 5$

$$1. (F \vee G) \leftrightarrow H$$

$$2. (\neg G \wedge H) \wedge (\neg K \vee H)$$

$$3. (F \vee G) \rightarrow K$$

$$\therefore K$$

1	$(F \vee G) \leftrightarrow H$	Pr.
2	$(\neg G \wedge H) \wedge (\neg K \vee H)$	Pr.
3	$(F \vee G) \rightarrow K$	Pr.
<b>WTS:</b> $K$		
4	$\neg G \wedge H$	$\wedge E, 2$
5	$\neg K \vee H$	$\wedge E, 2$
6	$H$	$\wedge E, 4$
7	$F \vee G$	$\leftrightarrow E, 1, 6$
8	$K$	$\rightarrow E, 3, 7$

1.  $X \wedge Y$

$\therefore (Y \wedge X) \wedge (X \wedge X)$

1	<u><math>X \wedge Y</math></u>	Pr.
<b>WTS:</b> $(Y \wedge X) \wedge (X \wedge X)$		
2	$X$	$\wedge E, 1$
3	$Y$	$\wedge E, 1$
4	$X$	Reit., 3
5	$X \wedge X$	$\wedge I, 2, 4$
6	$Y \wedge X$	$\wedge I, 2, 3$
7	$(Y \wedge X) \wedge (X \wedge X)$	$\wedge I, 5, 6$

1.  $X \wedge Y$

$\therefore (Y \wedge X) \vee Z$

1	<u><math>X \wedge Y</math></u>	Pr.
<b>WTS:</b> $(Y \wedge X) \vee Z$		
2	$X$	$\wedge E, 1$
3	$Y$	$\wedge E, 1$
4	$Y \wedge X$	$\wedge I, 2, 3$
5	$(Y \wedge X) \vee Z$	$\vee I, 4$

## 2 Homework 4

### 2.1 Chapter 15 - Block A

First  
Incorrect  
"Proof":

1	<u><math>(\neg L \wedge A) \vee L</math></u>	Pr.																					
2	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 10px;">2</td> <td style="border-left: 1px solid black; padding-left: 10px;"><u><math>\neg L \wedge A</math></u></td> <td style="padding-left: 10px;">Ass.</td> </tr> <tr> <td style="padding-right: 10px;">3</td> <td style="border-left: 1px solid black; padding-left: 10px;"><math>\neg L</math></td> <td style="padding-left: 10px;"><math>\wedge E, 2</math></td> </tr> <tr> <td style="padding-right: 10px;">4</td> <td style="border-left: 1px solid black; padding-left: 10px;"><math>A</math></td> <td style="padding-left: 10px;"><math>\wedge E, 2</math></td> </tr> <tr> <td style="padding-right: 10px;">5</td> <td style="border-left: 1px solid black; padding-left: 10px;"><u><math>L</math></u></td> <td style="padding-left: 10px;">Ass.</td> </tr> <tr> <td style="padding-right: 10px;">6</td> <td style="border-left: 1px solid black; padding-left: 10px;"><math>\perp</math></td> <td style="padding-left: 10px;"><math>\neg E, 3, 5</math> (Can't reference lines inside a closed sub-proof!)</td> </tr> <tr> <td style="padding-right: 10px;">7</td> <td style="border-left: 1px solid black; padding-left: 10px;"><math>A</math></td> <td style="padding-left: 10px;">Ex, 6</td> </tr> <tr> <td style="padding-right: 10px;">8</td> <td style="border-left: 1px solid black; padding-left: 10px;"><math>A</math></td> <td style="padding-left: 10px;"><math>\vee E, 1, 2-4, 5-7</math></td> </tr> </table>	2	<u><math>\neg L \wedge A</math></u>	Ass.	3	$\neg L$	$\wedge E, 2$	4	$A$	$\wedge E, 2$	5	<u><math>L</math></u>	Ass.	6	$\perp$	$\neg E, 3, 5$ (Can't reference lines inside a closed sub-proof!)	7	$A$	Ex, 6	8	$A$	$\vee E, 1, 2-4, 5-7$	
2	<u><math>\neg L \wedge A</math></u>	Ass.																					
3	$\neg L$	$\wedge E, 2$																					
4	$A$	$\wedge E, 2$																					
5	<u><math>L</math></u>	Ass.																					
6	$\perp$	$\neg E, 3, 5$ (Can't reference lines inside a closed sub-proof!)																					
7	$A$	Ex, 6																					
8	$A$	$\vee E, 1, 2-4, 5-7$																					

Second  
Incorrect  
"Proof":

1	$A \wedge (B \wedge C)$	Pr.
2	$(B \vee C) \rightarrow D$	Pr.
3	$B \wedge C$	$\wedge E, 1$
3.5	$B$	$\wedge E, 3$
4	$B \vee C$	$\vee I, 3.5$
5	$D$	$\rightarrow E, 4, 2, 4$ (but zee's chill abt line-number ordering)

## 2.2 Chapter 15 - Block B

1	$P \wedge S$	Pr.
2	$S \rightarrow R$	Pr.
3	$P$	$\wedge E, 1$
4	$S$	$\wedge E, 1$
5	$R$	$\rightarrow E, 2, 4$
6	$R \vee E$	$\vee I, 5$

Corresponding Argument:  
 $P \wedge S, S \rightarrow R \therefore R \vee E$

1	$A \rightarrow D$	Pr.
2	$A \wedge B$	Ass.
3	$A$	$\wedge E, 2$
4	$D$	$\rightarrow E, 1, 3$
5	$D \vee E$	$\vee I, 4$
6	$(A \wedge B) \rightarrow (D \vee E)$	$\rightarrow I, 2-5$

Corresponding Argument:  
 $A \rightarrow D \therefore (A \wedge B) \rightarrow (D \vee E)$

1	$\neg L \rightarrow (J \vee L)$	Pr.
2	$\neg L$	Pr.
3	$J \vee L$	$\rightarrow E, 1, 2$
4	$J$	Ass.
5	$J \wedge J$	$\wedge I, 4, 4$
6	$J$	$\wedge E, 5$ or Reit., 4
7	$L$	Ass.
8	$\perp$	$\neg E, 7, 2$
9	$J$	Ex., 8
10	$J$	$\vee E, 3, 4-6, 7-9$

Corresponding Argument:  $\neg L \rightarrow (J \vee L), \neg L \therefore J$

### 2.3 Chapter 15 Block C, Proofs 1–4

1.  $J \rightarrow \neg J \therefore \neg J$

1	$J \rightarrow \neg J$	Pr.
<b>WTS:</b> $\neg J$		
2	$J$	Ass.
3	$\neg J$	$\rightarrow E, 1, 2$
4	$\perp$	$\neg E, 2, 3$
5	$\neg J$	$\neg I, 2-4$

2.  $Q \rightarrow (Q \wedge \neg Q) \therefore \neg Q$

1		<u><math>Q \rightarrow (Q \wedge \neg Q)</math></u>	Pr.	
<b>WTS:</b> $\neg Q$				
2			<u><math>Q</math></u>	Ass.
3			$Q \wedge \neg Q$	$\rightarrow E, 1, 2$
4			$\neg Q$	$\wedge E, 3$
5			$\perp$	$\neg E, 3, 4$
6			$\neg Q$	$\neg I, 2-5$

3.  $A \rightarrow (B \rightarrow C) \therefore (A \wedge B) \rightarrow C$

1		<u><math>A \rightarrow (B \rightarrow C)</math></u>	Pr.	
<b>WTS:</b> $(A \wedge B) \rightarrow C$				
2			<u><math>A \wedge B</math></u>	Ass.
3			$A$	$\wedge E, 2$
4			$B \rightarrow C$	$\rightarrow E, 1, 3$
5			$B$	$\wedge E, 2$
6			$C$	$\rightarrow E, 4, 5$
7			$(A \wedge B) \rightarrow C$	$\rightarrow I, 2-6$

4.  $K \wedge L \therefore K \leftrightarrow L$

1		<u><math>K \wedge L</math></u>	Pr.	
<b>WTS:</b> $K \leftrightarrow L$				
2			<u><math>K</math></u>	Ass.
3			$L$	$\wedge E, 1$
4			<u><math>L</math></u>	Ass.
5			$K$	$\wedge E, 1$
6			$K \leftrightarrow L$	$\leftrightarrow I, 2-3, 4-5$

### 3 Homework 5

#### 3.1 Chapter 16, Block A, Question 1

1.  $A \rightarrow B, A \rightarrow C \therefore A \rightarrow (B \wedge C)$

1		$A \rightarrow B$	Pr.
2		$A \rightarrow C$	Pr.
<b>WTS:</b> $A \rightarrow (B \wedge C)$			
3			$A$ Ass.
4			$B$ $\rightarrow$ E, 1, 3
5			$C$ $\rightarrow$ E, 2, 3
6			$B \wedge C$ $\wedge$ I, 4, 5
7		$A \rightarrow (B \wedge C)$	$\rightarrow$ I, 3-6

#### 3.2 Chapter 16, Block D, Question 1

**WTS:**  $\neg\neg A \rightarrow A$

1			$\neg\neg A$ Ass.	
2				$\neg A$ Ass.
3				$\perp$ $\neg$ E, 1, 2
4		$A$	IP, 2-3	
5		$\neg\neg A \rightarrow A$	$\rightarrow$ I, 1-4	



### 3.3 Chapter 17, Block A, Fill-in Proofs 1 and 2

1	$W \rightarrow \neg B$	Pr.		1	$L \leftrightarrow \neg O$	Pr.
2	$A \wedge W$	Pr.		2	$L \vee \neg O$	Pr.
3	$B \vee (J \wedge K)$	Pr.		3	$\neg L$	Ass.
4	$W$	$\wedge E, 2$		4	$\neg O$	$DS, 2, 3$
5	$\neg B$	$\rightarrow E, 1, 4$		5	$L$	$\leftrightarrow E, 1, 4$
6	$J \wedge K$	$DS, 3, 5$		6	$\perp$	$\neg E, 5, 3$
7	$K$	$\wedge E, 6$		7	$\neg\neg L$	$\neg I, 3-6$
				8	$L$	$DNE, 7$ or $IP, 3-6$

### 3.4 Chapter 17, Block B, Question 1

1.  $E \vee F, F \vee G, \neg F \therefore E \wedge G$

1	$E \vee F$	Pr.
2	$F \vee G$	Pr.
3	$\neg F$	Pr.
<b>WTS:</b>	$E \wedge G$	
4	$E$	$DS 1, 3$
5	$G$	$DS 2, 3$
6	$E \wedge G$	$\wedge I, 4, 5$