Class 5: Translations in TFL

LOGIC

ZEE PERRY

5-C

1. Ava and Harrison are both electricians.

2. If Ava is a firefighter, then she is satisfied with her career.

3. Ava is a firefighter, unless she is an electrician.

5. Neither Ava nor Harrison is an electrician.

7. Harrison is satisfied only if he is a firefighter.

9. Ava is satisfied with her career if and only if Harrison is not satisfied with his.

11. It cannot be that Harrison is both an electrician and a firefighter.

Symbolization Key

E_1: Ava is an electrician.

E_2: Harrison is an electrician.

F_1: Ava is a firefighter.

F_2: Harrison is a firefighter.

S_1: Ava is satisfied with her career.

S_2: Harrison is satisfied with his career.

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1. $(E_1 \wedge E_2)$

$\wedge$ = and

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2. $(F_1 \rightarrow S_1)$

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2. $(F_1 \rightarrow S_1)$

3. $(F_1 \vee E_1)$

$(\neg F_1 \rightarrow E_1)$

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$V = 6r$

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7. $(S_2 \rightarrow F_2)$

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2. $(F_1 \rightarrow S_1)$

3. $(F_1 \vee E_1)$

5. $(\neg E_1 \wedge \neg E_2)$

7. $(S_2 \rightarrow F_2)$

9. $(S_1 \leftrightarrow \neg S_2)$

11. $\neg(E_2 \wedge F_2)$

5-E

1. Alice and Bob are both spies.

2. If either Alice or Bob is a spy, then the code has been broken.

3. If neither Alice nor Bob is a spy, then the code remains unbroken.

4. The German embassy will be in an uproar, unless someone has broken the code.

5. Either the code has been broken or it has not, but the German embassy will be in an uproar regardless.

6. Either Alice or Bob is a spy, but not both.

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B: Bob is a spy
C: Code has been broken
D: German Embassy will be in an uproar
E: Code remains unbroken

1.
2.
3.
4.
5.
6.

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2.

3.

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D: German Embassy will be in an uproar

1. $(A \wedge B)$

2. $((A \vee B) \rightarrow C)$

3. $(\neg(A \vee B) \rightarrow \neg C)$

4. $(D \vee C)$

5.

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2. $((A \vee B) \rightarrow C)$

3. $(\neg(A \vee B) \rightarrow \neg C)$

4. $(\neg C \rightarrow D)$

5.

6.

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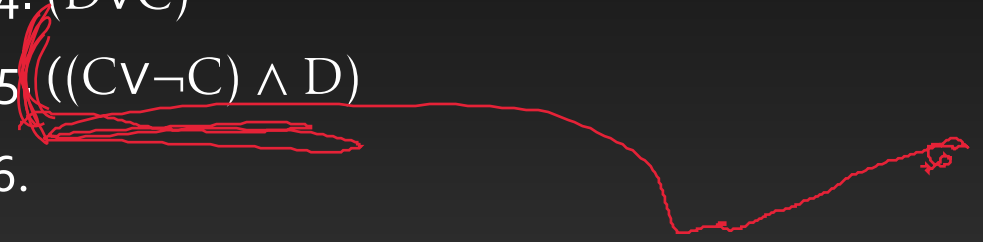
1. $(A \wedge B)$
2. $((A \vee B) \rightarrow C)$
3. $(\neg(A \vee B) \rightarrow \neg C)$
4. $(D \vee C)$
5. $((C \vee \neg C) \wedge D)$
6.



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2. $((A \vee B) \rightarrow C)$

3. $(\neg(A \vee B) \rightarrow \neg C)$

4. $(D \vee C)$

5. $(C \vee \neg C) \wedge D)$

6. $((A \vee B) \wedge \neg(A \wedge B))$

Review of Previous Class: Arguments

What is a **sentence**, in logic?

Sentences are *full declarative statements*

i.e. they can be true/false

What are **arguments**, in logic?

Arguments are collections of (any number of) sentences

- with ***one*** sentence designated the "**conclusion**"
- and ***all other*** sentences called the "**premises**"

What is **the purpose of an argument**, in logic?

Arguments are supposed to provide *rationally compelling* reason to accept their conclusion.

The premises are supposed to ***support*** the conclusion.

Review of Previous Class: Valid arguments hold in "every case"

If an argument is **Valid**, its premises imply its conclusion no matter what.

- You can dream up any scenario you want, and, so long as the premises are true in the scenario, then the conclusion *must* be true (assuming the argument is valid)

Invalid arguments have Counterexamples.

- Counterexamples are: "cases" that show the argument is bad/invalid.
- Where a "case" is a possible hypothetical scenario (*waves hands*)

Review of Previous Class:
Logical Consequence

A sentence, call it "C", is a **logical consequence** of a list of other sentences, call them "P_1", "P_2", "P_3" (and so on until "P_n"), if and only if there is **no case** where P_1, P_2, …. , P_n are all true and C is not true.

An argument is **Valid** if its conclusion is a **logical consequence** of its premises.

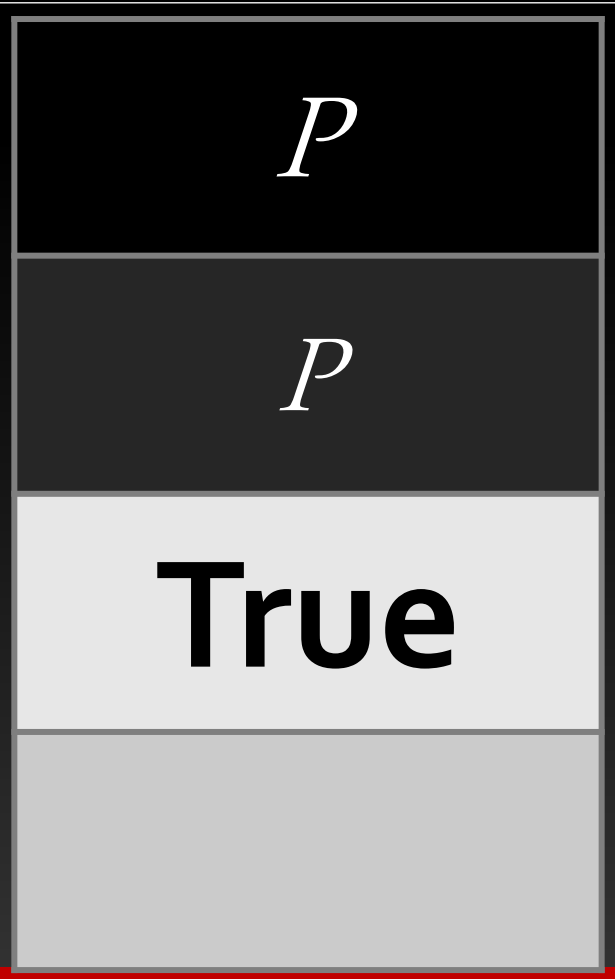
C	is a **(logical) consequence** of	P_1, P_2, …. , and P_n
C	**follows from**	P_1, P_2, …. , and P_n
P_1, P_2,… and P_n	**entail**	C

Truth-Functional Logic (TFL)

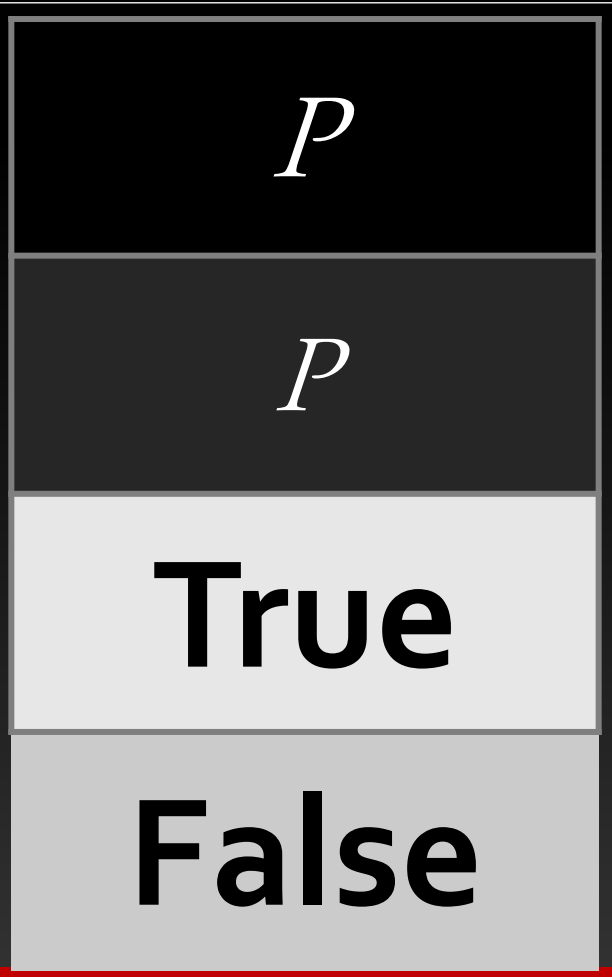
	Sentence Component	Representation in English (a popular Natural Language)	Representation in TFL
SENTENCES	Atomic Sentences	"I love to eat pizza" "Electrons are point-particles" (like, could be *literally any sentence*)	$A, B, C, D, E, F, G, H, \ldots$
SENTENTIAL CONNECTIVES	Negation	"not..", "it's not the case that...", "it's not true that..."	$\neg$ (Alternatively, '~')
	Conjunction	".. and..", ".. but..", ".. however .."	$\wedge$ (Alternatively, '&', '▪')
	Disjunction	"Either... or..", ".. or..",	$\vee$
	Material Conditional	"if.. then.." , ".. only if .." (or ".. if ..", but only in reverse!)	$\rightarrow$ (Alternatively, '⊃')
	Bi-Conditional	".. if and only if..", ".. iff..", "... just in case..",	$\leftrightarrow$ (Alternatively, '≡')

Sentences have Truth-Values

Sentences have Truth-Values



There are only TWO Truth-Values



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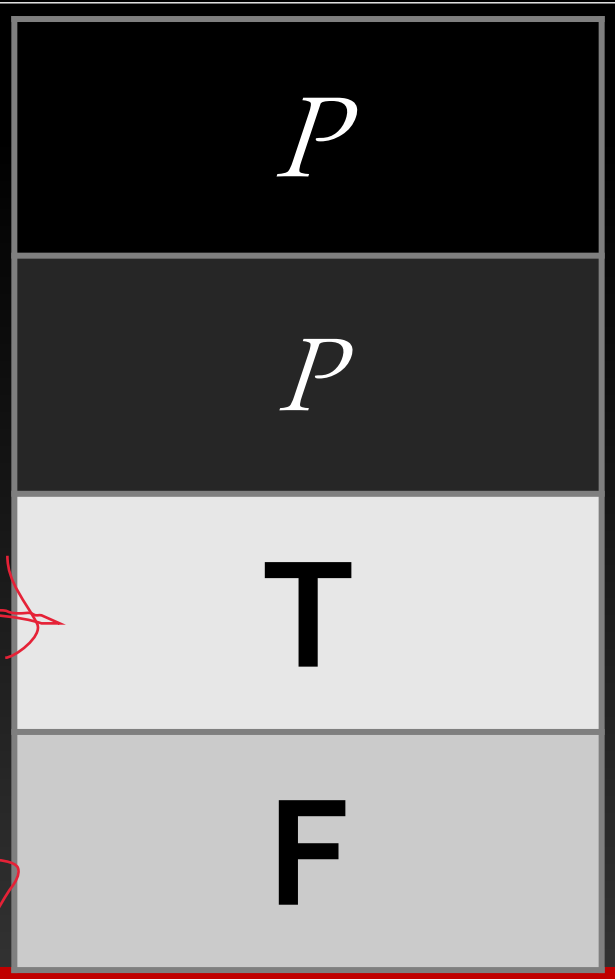

Possible Scenario: P is true

Poss. Scenario: P is false

Negation "flips" Truth-Value

$(\neg Q)$	
$\neg Q$	Q

Negation "flips" Truth-Value

$(\neg Q)$	
$\neg Q$	Q
	T

Negation "flips" Truth-Value

$(\neg Q)$	
$\neg Q$	Q
F	T

Negation "flips" Truth-Value

$(\neg Q)$	
$\neg Q$	Q
F	T
	F

Negation "flips" Truth-Value

$(\neg Q)$	
$\neg Q$	Q
F	T
T	F

Aside: Remember “cases”?

When talking about validity and logical consequence, we talked a lot about “cases”, which were something kinda like “possible, hypothetical scenarios”.

Now that we’re in TFL, the language is simple enough that we can make the concept of a “case” perfectly precise!

A case is **any** distribution of truth-values to each of the ***atomic sentences***.

Aside: Remember “cases”?

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Aside: Remember "cases"?

When talking about validity and logical consequence, we talked a lot about "cases", which were something kinda like "possible, hypothetical scenarios".

Now that we're in TFL, the language is simple enough that we can make the concept of a "case" perfectly precise!

- A case (in TFL) is **any** distribution of truth-values to **each** of the ***atomic sentences*** (in whatever argument/sentence you're considering)

Negation "flips" Truth-Value

$(\neg Q)$	
$\neg Q$	Q
F	T
T	F

Negation "flips" Truth-Value

Two possible cases:

$(\neg Q)$	
$\neg Q$	Q
Q is true	F	T
Q is false	T	F

Always start with the Atomic Sentences

$(A \wedge B)$		
A	$A \wedge B$	B

Always start with the Atomic Sentences

$(A \wedge B)$		
A	$A \wedge B$	B
T		
F		

What are all the possible truth-values for “A” and “B”?

$(A \wedge B)$		
A	$A \wedge B$	B
T		?
F		?

What are all the possible truth-values for “A” and “B”?

$(A \wedge B)$		
A	$A \wedge B$	B
T		?
F		?
?		?
?		?

What are all the possible truth-values for “A” and “B”?

$(A \wedge B)$		
A	$A \wedge B$	B
T		
F		

What are all the possible truth-values for “A” and “B”?

$(A \wedge B)$		
A	$A \wedge B$	B
T		T
F		

What are ALL the possible truth-values for “A” and “B”?

$(A \wedge B)$		
A	$A \wedge B$	B
T		T
F		

What are ALL the possible truth-values for “A” and “B”?

$(A \wedge B)$		
A	$A \wedge B$	B
T		T
F		
T		F

What are ALL the possible truth-values for “A” and “B”?

$(A \wedge B)$		
A	$A \wedge B$	B
T		T
F		T
T		F

What are ALL the possible truth-values for “A” and “B”?

$(A \wedge B)$		
A	$A \wedge B$	B
T		T
F		T
T		F
F		F

What are ALL the possible truth-values for "A" and "B"?

Four possible cases:

$(A \wedge B)$		
A	$A \wedge B$	B
T		T
F		T
T		F
F		F

Both A and B are true

B is true but A is false

A is true but B is false

Both A and B are false

What are ALL the possible truth-values for "A" and "B"?

Four possible cases:

	$(A \wedge B)$		
	A	$A \wedge B$	B
Both A and B are true	T		T
B is true but A is false	F		T
A is true but B is false	T		F
Both A and B are false	F		F

Quizlet Q~~1~~3:

In which "cases" is: "$(A \wedge B)$" a **TRUE** sentence?

What are ALL the possible truth-values for "A" and "B"?

Four possible cases:

	$(A \wedge B)$		
	A	$A \wedge B$	B
Both A and B are true	T		T
B is true but A is false	F		T
A is true but B is false	T		F
Both A and B are false	F	F	F

Quizlet Q3:

In which "cases" is: "$(A \wedge B)$" a **TRUE** sentence?

What are ALL the possible truth-values for "A" and "B"?

Four possible cases:

$(A \wedge B)$		
A	$A \wedge B$	B
T		T
F	F	T
T	F	F
F	F	F

Both A and B are true

B is true but A is false

A is true but B is false

Both A and B are false

Quizlet Q3:

In which "cases" is: "$(A \wedge B)$" a **TRUE** sentence?

What are ALL the possible truth-values for "A" and "B"?

Four possible cases:

$(A \wedge B)$		
A	$A \wedge B$	B
T	T	T
F	F	T
T	F	F
F	F	F

Both A and B are true

B is true but A is false

A is true but B is false

Both A and B are false

Quizlet Q3:

In which "cases" is: "$(A \wedge B)$" a **TRUE** sentence?

$(A \vee B)$		
A	$A \vee B$	B

$(A \vee B)$		
A	$A \vee B$	B
T		
F		

$(A \vee B)$		
A	$A \vee B$	B
T		T
F		T

$(A \vee B)$		
A	$A \vee B$	B
T	T	T
F	T	T
T	T	F
F	F	F

Four possible cases:

	$(A \vee B)$		
	A	$A \vee B$	B
Both A and B are true	T	T	T
B is true but A is false	F	T	T
A is true but B is false	T	T	F
Both A and B are false	F	F	F

$(A \rightarrow B)$		
A	$A \rightarrow B$	B

$(A \rightarrow B)$		
A	$A \rightarrow B$	B
T		
F		

$(A \rightarrow B)$		
A	$A \rightarrow B$	B
T		T
F		T

$(A \rightarrow B)$		
A	$A \rightarrow B$	B
T	T	T
F		T
T	F	F
F		F

Four possible cases:

	$(A \rightarrow B)$		
	A	$A \rightarrow B$	B
Both A and B are true	T		T
B is true but A is false	F		T
A is true but B is false	T		F
Both A and B are false	F		F

Four possible cases:

Both A and B are true

B is true but A is false

A is true but B is false

Both A and B are false

$(\ A \rightarrow B\ )$		
A	$A \rightarrow B$	B
T		T
F		T
T		F
F		F

Quizlet Q4:

In which "cases" is:
"$(\ A \rightarrow B\ )$"
a **TRUE** sentence?

Four possible cases:

	$(A \rightarrow B)$		
	A	$A \rightarrow B$	B
Both A and B are true	T	T	T
B is true but A is false	F		T
A is true but B is false	T		F
Both A and B are false	F		F

Four possible cases:

	$(A \rightarrow B)$		
	A	$A \rightarrow B$	B
Both A and B are true	T	T	T
B is true but A is false	F		T
A is true but B is false	T	F	F
Both A and B are false	F		F

Four possible cases:

$(A \rightarrow B)$		
A	$A \rightarrow B$	B
T	T	T
F	T	T
T	F	F
F	T	F

Both A and B are true

B is true but A is false

A is true but B is false

Both A and B are false

$(A \leftrightarrow B)$		
A	$A \leftrightarrow B$	B

$(\ A \leftrightarrow B\ )$		
A	$A \leftrightarrow B$	B
T		
F		

$(A \leftrightarrow B)$		
A	$A \leftrightarrow B$	B
T		T
F		T
T		F
F		F

Four possible cases:

	$(A \leftrightarrow B)$		
	A	$A \leftrightarrow B$	B
Both A and B are true	T		T
B is true but A is false	F		T
A is true but B is false	T		F
Both A and B are false	F		F

Four possible cases:

$(A \leftrightarrow B)$		
A	$A \leftrightarrow B$	B
T	T	T
F		T
T		F
F	T	F

Both A and B are true

B is true but A is false

A is true but B is false

Both A and B are false

Four possible cases:

$(A \leftrightarrow B)$		
A	$A \leftrightarrow B$	B
T	T	T
F	F	T
T	F	F
F	T	F

Both A and B are true

B is true but A is false

A is true but B is false

Both A and B are false