

Class 9 Proof Theory, continued

Zee R. Perry

Let's do some logic

NO QUIZ LET

Natural Deduction is about..

Arguments and Proofs

In a Natural Deduction system, we're able to show that certain arguments are valid by **constructing proofs** in the right way.

Specifically, a well-constructed proof shows that the argument can be broken down into a series of **simple, valid arguments**. And so the argument as a whole must be valid, since it relies on a valid argument at every step!

Natural deduction is a theory of *formal proofs*

Constructing a formal proof is a step-by-step process, and each step must be performed in accordance with the rules of a Natural Deduction system.

How to Build a Proof in Three Easy Steps:

The rules of natural deduction are rules on **HOW TO MAKE PROOFS** (we'll be formatting our proofs in what's called the "Fitch style", but the name isn't important)

General Rules: (These apply at every step)

- **Numbered lines:** Every line in the proof is numbered. (So every time you make a new line, it should get a number.)
- **Vertical stroke:** For the entire proof, draw a vertical stroke to the right of the line number, and continue this stroke with each new line.

Step A: Premises

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Step A: Premises

Step B: Apply Rules

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Step A: Premises

Step B: Apply Rules

Step C: Conclusion

How to Build a Proof in Three Easy Steps:

General Rules: (These apply at every step)

- **Numbered lines:** Every line in the proof is numbered.
- **Vertical stroke:** For the *entire proof*, draw a vertical stroke to the right of the line number.

Step A: Premises - List all the premises.

- **Line:** List each premise on its own line.
- **Format:** Number each line you add. Draw the vertical stroke.
- **Right-Margin:** On each line with a premise, write “Pr.” on the far right-hand side of the proof.
- **At the end:** Draw a horizontal line under the final premise’s line.

How to Build a Proof in Three Easy Steps: Step B

Step B: Apply Rules - Add new lines according to rules.

- **Line:** Use a rule to determine what new lines you are allowed to add, given what's come before.
- **Format:** Number each line you add. Draw the vertical stroke. *In addition*, include any other formatting as required by the rules applied in this line or previous ones.
- **Right-Margin:** Write the rule that you used to generate that line, and the numbers of the lines the rule applied to.

How to Build a Proof in Three Easy Steps: Step C

Step C: Conclusion - Step B ends when you reach the conclusion

- **Line:** If, during Step B, you manage to write down the conclusion, still following the rules of Natural Deduction, then you're done! ***Congrats!!***
- **Format:** Number each line you add. Draw the vertical stroke.
- **Right-Margin:** Write the rule that you used to generate that line, and the numbers of the lines the rule applied to.

Rule: Reiteration (“Reit.” or “R”)

The idea is this: The following argument-form is always valid, no matter what sentence you use.

$$\frac{1. P}{\therefore P}$$

$$\frac{1. (A \leftrightarrow G)}{\therefore (A \leftrightarrow G)}$$

$$\frac{1. \text{I'm busy}}{\therefore \text{I'm busy}}$$

Rule: Reiteration (“Reit.” or “R”)

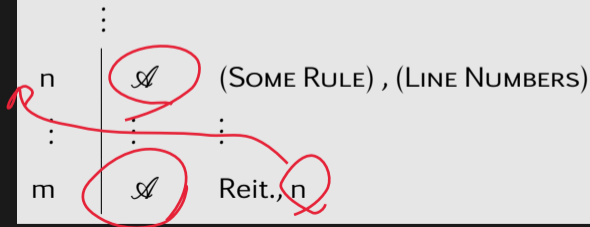
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Reiteration Rule, “Reit.”



Rule: Conjunction Introduction (“ $\wedge I$ ” or “ $\wedge In$ ”)

1. P
2. $Q \vee P$

 $\therefore P \wedge (Q \vee P)$

1. $(A \leftrightarrow G)$
2. $\neg S$

 $\therefore (A \leftrightarrow G) \wedge \neg S$

1. There's a giraffe in this building
2. NYU Shanghai is in China

 \therefore There's a giraffe in this building *and*
~~NY~~ Shanghai is in China

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Conjunction Introduction, “ $\wedge I$ ”

n	\mathcal{A}	(SOME RULE), (LINE NUMBERS)
:	:	:
m	\mathcal{B}	(SOME RULE), (LINE NUMBERS)
:	:	:
u	$\mathcal{A} \wedge \mathcal{B}$	$\wedge I$, n, m

Rule: Conjunction Elimination (“ $\wedge E$ ” or “ $\wedge Out$ ”)

$$\frac{1. P \wedge (Q \vee P)}{\therefore Q \vee P}$$

$$\frac{1. \neg S \wedge (A \leftrightarrow G)}{\therefore \neg S}$$

1. There's a giraffe in this building
and NYC is in NY

 \therefore There's a giraffe in this building

Rule: Conjunction Elimination (“ $\wedge E$ ” or “ $\wedge Out$ ”)

$$\frac{1. P \wedge (Q \vee P)}{\therefore \text{~~Q \vee P~~}}$$

$$\therefore P$$

$$\frac{1. \neg S \wedge (A \leftrightarrow G)}{\therefore \text{~~\neg S~~}}$$

$$\therefore (A \leftrightarrow G)$$

1. There's a giraffe in this building
and NYC is in NY

\therefore There's a giraffe in this building

Conjunction Elimination, “ $\wedge E$ ”

n | $A \wedge B$ (SOME RULE), (LINE NUMBERS)

:

:

:

u | A

A

$\wedge E$

,

n

:

:

:

v | B

B

$\wedge E$

,

n

Rule: Disjunction Introduction (“ $\vee I$ ” or “ $\vee In$ ”)

Okay, hear me out. This is gonna seem **WILD**. Disjunction introduction is the reasoning involved in the following arguments (all of which are **valid!**):

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1. It's Wednesday.

\therefore Either it's Wednesday or
I'm a kettle of fish.

1. $\neg S \wedge A$

$\therefore (\neg S \wedge A) \vee B$

1. P

$\therefore P \vee Q$

Rule: Disjunction Introduction (“ $\vee I$ ” or “ $\vee In$ ”)

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 $\therefore P \vee Q$

Disjunction Introduction, “ $\vee I$ ”

Variable for any sentence in TFL

For ANY “ B ”!

n		A	(SOME RULE), (LINE NUMBERS)
:		:	:
u		$A \vee B$	$\vee I$, n

Rule: Conditional Elimination (“ \rightarrow E” or “ \rightarrow Out”)

Conditional Elimination works exactly as you'd expect it would. If you have a conditional on some line, and the antecedent of that conditional on another line, then you can write down the consequent of that conditional.

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1. If it's raining then my hair will get wet.

2. It's raining.

\therefore My hair will get wet

1. $(\neg S \wedge A) \rightarrow (B \leftrightarrow A)$

2. $\neg S \wedge A$

$\therefore B \leftrightarrow A$

1. $P \rightarrow Q$

2. P

$\therefore Q$

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1. $P \rightarrow Q$

2. P

$\therefore Q$

Conditional Elimination, “ \rightarrow E”

n | $A \rightarrow B$ (SOME RULE), (LINE NUMBERS)

⋮

⋮

⋮

m | A (SOME RULE), (LINE NUMBERS)

⋮

⋮

⋮

u | B \rightarrow E, n,m

Proof without Rules & Line Numbers

1	$P \wedge S$	_____ , _____
2	$S \rightarrow R$	_____ , _____
WTS:	$R \vee E$	
3	P	_____ , _____
4	S	_____ , _____
5	R	_____ , _____
6	$R \vee E$	_____ , _____

Proof without TFL Sentences after Premises.

1	$(Q \wedge P) \rightarrow S$	Pr.
2	$P \wedge Q$	Pr.
WTS:	$S \wedge Q$	
3	_____	$\wedge E, 2$
4	_____	$\wedge E, 2$
5	_____	$\wedge I, 3, 4$
6	_____	$\rightarrow E, 1, 5$
7	_____	$\wedge I, 4, 6$

New Rules: Adding “Layers” to our proofs!

The remaining rules in our Natural Deduction system for TFL will involve Assumptions and Sub-proofs.

These rules will indicate when you are allowed to start a *new* proof **within** your current proof.

The simplest form of this comes from Conditional Introduction (“ \rightarrow I”).

Rule: Conditional Introduction (“ \rightarrow I” or “ \rightarrow In”)

Conditional Introduction is a rule that lets you write down a **conditional** statement (i.e. a sentence whose main connective is ‘ \rightarrow ’), given some lines earlier in your proof. What would this even look like? What—*IN GENERAL*—is required for “ $P \rightarrow Q$ ” to be true?

Rule: Conditional Introduction (“ \rightarrow I” or “ \rightarrow In”)

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What—*IN GENERAL*—is required for “ $P \rightarrow Q$ ” to be true?

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But it also *doesn't require P to actually be true!* So how could we make sense of this in our proof system?

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But how could we—while in the middle of a proof!—show that some *other* argument is valid??

Simplest Solution: Put a proof on your proof so you can prove while you prove! So how the heck would *that* work?!

Sub-proofs and Assumptions

Within a proof, you can start **another** proof once you've moved past the premises stage:

1 | $A \rightarrow C$ Pr.

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WTS:	$A \rightarrow B$	

WANT TO SHOW

Sub-proofs and Assumptions

When you start a sub-proof, you introduce *Assumptions* instead of premises.

1		$A \rightarrow C$	Pr.
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WTS:		$A \rightarrow B$	
3			

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3			
		A	Ass.
		└──┘	

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Any line you add *within* your sub-proof can reference any line above it, *including lines from your main proof!*

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2		$C \rightarrow B$	Pr.
<hr/>			
WTS:		$A \rightarrow B$	
3			Ass.
		A	
4			
		<hr/>	



Sub-proofs and Assumptions

1		$A \rightarrow C$	Pr.
2		$C \rightarrow B$	Pr.
<hr/>			
WTS:		$A \rightarrow B$	
3			Ass.
		A	
4			
		C	$\rightarrow E, 1, 3$

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Sub-proofs and Assumptions

1		$A \rightarrow C$	Pr.
2		$C \rightarrow B$	Pr.

WTS: $A \rightarrow B$

3			A	Ass.	But it doesn't work in reverse! You <i>cannot</i> reference any line from the sub-proof after you've exited and returned to the main proof!
4			C	$\rightarrow E, 1, 3$	
5			B	$\rightarrow E, 2, 4$	
6					

Sub-proofs and Assumptions

1		$A \rightarrow C$	Pr.
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WTS: $A \rightarrow B$

3			A	Ass.
4			C	$\rightarrow E, 1, 3$
5			B	$\rightarrow E, 2, 4$
6			B	Reit, 5

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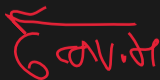
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 \rightarrow *conv. it*

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However, there are some rules that can only be applied *if a sub-proof has occurred earlier in the proof!* One such rule is $\rightarrow I$

Sub-proofs and Assumptions

1		$A \rightarrow C$	Pr.
2		$C \rightarrow B$	Pr.
<hr/>			
WTS:		$A \rightarrow B$	
3			Ass.
4			$\rightarrow E, 1, 3$
5			$\rightarrow E, 2, 4$
6			$\rightarrow I, 3-5$

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<hr/>			
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3			
		A	Ass.
<hr/>			
4		C	$\rightarrow E, 1, 3$
5		B	$\rightarrow E, 2, 4$
6		$A \rightarrow B$	$\rightarrow I, 3-5$

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Rule: Conditional Introduction (“ $\rightarrow I$ ” or “ $\rightarrow In$ ”)

Conditional Introduction appeals to the following kind of reasoning:

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Since $\frac{1. P}{\therefore Q}$ is a valid argument, it must be true that $P \rightarrow Q$

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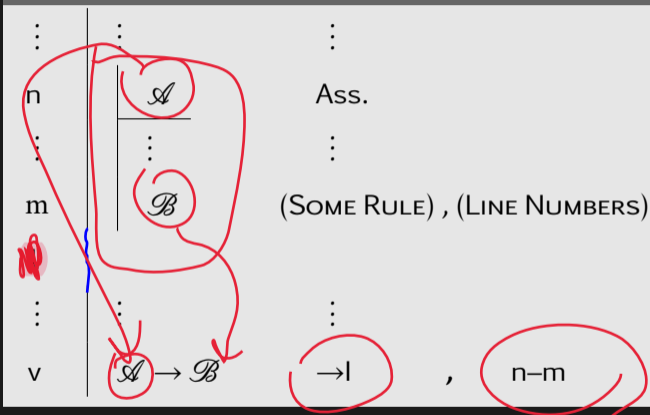
Conditional Introduction appeals to the following kind of reasoning:

Since $\frac{1. P}{\therefore Q}$ is a valid argument, it must be true that $P \rightarrow Q$

If we assume that it'll rain tonight, then it follows that I'll get my hair wet. So, *no matter what* the weather's like, we know that the conditional “If it rains, then I'll get my hair wet” is true.

Rule: Conditional Introduction (" $\rightarrow I$ " or " $\rightarrow I_n$ ")

Conditional Introduction, " $\rightarrow I$ "



Rule: Disjunction Elimination (“ $\vee E$ ” or “ $\vee Out$ ”)

Disjunction Elimination *also* relies on the use of sub-proofs!

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Disjunction Elimination *also* relies on the use of sub-proofs!

1. X is either in the library or in the student center.
2. If X is in the library, then X is on College Ave.
3. If X is in the student center, then X is on College Ave.

$\therefore X$ is on College Ave!

Rule: Disjunction Elimination ("VE" or "VOut")

Disjunction Elimination, "VE"

i	$A \vee B$	(SOME RULE), (LINE NUMBERS)
:	:	:
n	A	Ass.
m	C	(SOME RULE), (LINE NUMBERS)
:	:	:
u	B	Ass.
v	C	(SOME RULE), (LINE NUMBERS)
j	C	VE, i, n-m, u-v

Rule: Negation Elimination (“ $\neg E$ ” or “ ~~$\exists I$~~ ” or “ $\perp I$ ”)

Negation Elimination is also sometimes known as Contradiction Introduction, because it only “eliminates” negations in a special case, when its the negation of a sentence that appears elsewhere in your proof.

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Negation Elimination, “ $\neg E$ ”

\vdots	\vdots	\vdots
n	\mathcal{A}	(SOME RULE) , (LINE NUMBERS)
\vdots	\vdots	\vdots
m	$\neg \mathcal{A}$	(SOME RULE) , (LINE NUMBERS)
j	\perp	$\neg E$, n, m

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n	\cancel{X}	(SOME RULE) , (LINE NUMBERS)
\vdots	\vdots	\vdots
m	$\neg \cancel{X}$	(SOME RULE) , (LINE NUMBERS)
j	\cancel{X}	$\neg E$, n, m

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j	\perp	$\neg E$, n, m

Rule: Negation Introduction (“ $\neg I$ ” or “ $\neg In$ ”)

Negation introduction is a way of introducing a negation by “eliminating” a contradiction (when that contradiction arises **IN A SUB-PROOF!**). Specifically, if a sub-proof leads you to a contradiction, then you *know* that the assumptions you made *cannot* be true!

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Negation Introduction, “ $\neg I$ ”

\vdots	\vdots	\vdots
n	\mathcal{A}	Ass.
\vdots	\vdots	\vdots
m	\perp	(SOME RULE), (LINE NUMBERS)
j	$\neg\mathcal{A}$	$\neg I$, n-m

Rule: Indirect Proof (“IP”)

Indirect Proof, also called “Proof by Contradiction”, is extremely similar to negation introduction. However, instead of *introducing* a negation at the end, we’re *assuming* a negated statement, in order to show that this leads to a contradiction. If assuming that “ P ” is false leads to a contradiction, then it must* be that P is true!

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Indirect Proof, "IP"

⋮	⋮	⋮
n	$\neg \mathcal{A}$	Ass.
⋮	⋮	⋮
m	\perp	(SOME RULE), (LINE NUMBERS)
j	\mathcal{A}	IP, n-m

Rule: Explosion (“Ex” or “ \perp E” or ‘XOut’)

The Explosion rule has a fun name because it describes a really wild feature of TFL: Literally anything follows from a contradiction. That is, any argument with a contradiction among its premises is valid, no matter what else is in the argument!

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Explosion, “Ex”

n	\perp	(SOME RULE), (LINE NUMBERS)
:	\vdots	\vdots
m	\perp	Ex, n

\perp
—
p

Homework!

From Chapter 15

Block A: Correct both Proofs

Block B: Complete both Proofs

Block C: Do ONLY 1, 2, 3, 4