Class 9 Proof Theory, continued

Zee R. Perry

Let's do some logic

NO QUIZLET

Natural Deduction is about..

Arguments and **Proofs**

In a Natural Deduction system, we're able to show that certain arguments are valid by **constructing proofs** in the right way.

Specifically, a well-constructed proof shows that the argument can be broken down into a series of **simple, valid arguments**. And so the argument as a whole must be valid, since it relies on a valid argument at every step!

Natural deduction is a theory of *formal* proofs

Constructing a formal proof is a step-by-step process, and each step must be performed in accordance with the rules of a Natural Deduction system.

How to Build a Proof in Three Easy Steps:

The rules of natural deduction are rules on **HOW TO MAKE PROOFS** (we'll be formatting our proofs in what's called the "Fitch style", but the name isn't important)

General Rules: (These apply at every step)

- **Numbered lines:** Every line in the proof is numbered. (So every time you make a new line, it should get a number.)
- **Vertical stroke:** For the entire proof, draw a vertical stroke to the right of the line number, and continue this stroke with each new line.

Step A: Premises

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Step A: Premises

Step B: Apply Rules

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Step A: Premises

Step B: Apply Rules

Step C: Conclusion

How to Build a Proof in Three Easy Steps:

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- **Numbered lines:** Every line in the proof is numbered.
- **Vertical stroke:** For the *entire proof,* draw a vertical stroke to the right of the line number.

Step A: Premises - List all the premises.

- **Line:** List each premise on its own line.
- **Format:** Number each line you add. Draw the vertical stroke.
- **Right-Margin:** On each line with a premise, write "Pr." on the far right-hand side of the proof.
- **At the end:** Draw a horizontal line under the final premise's line.

How to Build a Proof in Three Easy Steps: **Step B**

Step B: Apply Rules - Add new lines according to rules.

- **Line:** Use a rule to determine what new lines you are allowed to add, given what's come before.
- **Format:** Number each line you add. Draw the vertical stroke. ***In addition,*** include any other formatting as required by the rules applied in this line or previous ones.
- **Right-Margin:** Write the rule that you used to generate that line, and the numbers of the lines the rule applied to.

How to Build a Proof in Three Easy Steps: **Step C**

Step C: Conclusion - Step B ends when you reach the conclusion

- **Line:** If, during Step B, you manage to write down the conclusion, still following the rules of Natural Deduction, then you're done! ***Congrats!!***
- **Format:** Number each line you add. Draw the vertical stroke.
- **Right-Margin:** Write the rule that you used to generate that line, and the numbers of the lines the rule applied to.

Rule: Reiteration ("Reit." or "R")

The idea is this: The following argument-form is always valid, no matter what sentence you use.

1. P	1. $(A \leftrightarrow G)$	1. I'm busy
$\therefore$ P	$\therefore$ $(A \leftrightarrow G)$	$\therefore$ I'm busy

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The idea is this: The following argument-form is always valid, no matter what sentence you use.

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Reiteration Rule, "Reit."

	$\vdots$	
n	$\mathscr{A}$	(Some Rule) , (Line Numbers)
$\vdots$	$\vdots$	$\vdots$
m	$\mathscr{A}$	Reit., n

Rule: Conjunction Introduction (“∧I” or “∧In”)

1. P
2. $Q \vee P$

$\therefore\ P \wedge (Q \vee P)$

1. $(A \leftrightarrow G)$
2. $\neg S$

$\therefore\ (A \leftrightarrow G) \wedge \neg S$

1. There’s a giraffe in this building
2. NYU Shanghai is in China

$\therefore$ There’s a giraffe in this building *and* NYU Shanghai is in China

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2. NYU Shanghai is in China

∴ There's a giraffe in this building *and* Shanghai is in China

Conjunction Introduction, "∧I"

n	$\mathscr{A}$	(Some Rule)	, (Line Numbers)
⋮	⋮	⋮	
m	$\mathscr{B}$	(Some Rule)	, (Line Numbers)
⋮	⋮	⋮	
u	$\mathscr{A} \wedge \mathscr{B}$	∧I	, n, m

Rule: Conjunction Elimination ("∧E" or "∧Out")

1. $P \wedge (Q \vee P)$

$\therefore\ Q \vee P$

1. $\neg S \wedge (A \leftrightarrow G)$

$\therefore\ \neg S$

1. There's a giraffe in this building *and* NYC is in NY

$\therefore$ There's a giraffe in this building

Rule: Conjunction Elimination ("∧E" or "∧Out")

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1. $\neg S \wedge (A \leftrightarrow G)$

$\therefore$ ~~$\neg S$~~ $(A \leftrightarrow G)$

1. There's a giraffe in this building *and* NYC is in NY

$\therefore$ There's a giraffe in this building

Conjunction Elimination, "∧E"		
n	$\mathscr{A} \wedge \mathscr{B}$	(SOME RULE) , (LINE NUMBERS)
⋮	⋮	⋮
u	$\mathscr{A}$	∧E , n
⋮	⋮	⋮
v	$\mathscr{B}$	∧E , n

Rule: Disjunction Introduction ("$\vee$I" or "$\vee$In")

Okay, hear me out. This is gonna seem **WILD**. Disjunction introduction is the reasoning involved in the following arguments (all of which are **valid!**):

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1. It's Wednesday.

$\therefore$ Either it's Wednesday or I'm a kettle of fish.

1. $\neg S \wedge A$

$\therefore$ $(\neg S \wedge A) \vee B$

1. P

$\therefore$ $P \vee Q$

Rule: Disjunction Introduction ("∨I" or "∨In")

1. It's Wednesday.

∴ Either it's Wednesday or I'm a kettle of fish.

1. $\neg S \wedge A$

∴ $(\neg S \wedge A) \vee B$

1. P

∴ $P \vee Q$

Rule: Disjunction Introduction ("∨I" or "∨In")

1. It's Wednesday.

∴ Either it's Wednesday or I'm a kettle of fish.

1. $\neg S \land A$

∴ $(\neg S \land A) \lor B$

1. P

∴ $P \lor Q$

Disjunction Introduction, "∨I"

Variable for any sentence in TFL

For **ANY** "$\mathscr{B}$" !

n	$\mathscr{A}$	(Some Rule) , (Line Numbers)	
⋮	⋮	⋮	
u	$\mathscr{A} \lor \mathscr{B}$	∨I	, n

Rule: Conditional Elimination ("→E" or "→Out")

Conditional Elimination works exactly as you'd expect it would. If you have a conditional on some line, and the antecedent of that conditional on another line, then you can write down the consequent of that conditional.

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1. If it's raining then my hair will get wet.
2. It's raining.

∴ My hair will get wet

1. $(\neg S \wedge A) \rightarrow (B \leftrightarrow A)$
2. $\neg S \wedge A$

$\therefore \; B \leftrightarrow A$

1. $P \rightarrow Q$
2. P

$\therefore \; Q$

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1. $P \rightarrow Q$
2. P

∴ Q

Conditional Elimination, “→E”

n	$\mathscr{A} \rightarrow \mathscr{B}$	(Some Rule), (Line Numbers)
⋮	⋮	⋮
m	$\mathscr{A}$	(Some Rule), (Line Numbers)
⋮	⋮	⋮
u	$\mathscr{B}$	→E, n,m

Proof without Rules & Line Numbers

1	$P \wedge S$	____ , ____
2	$S \rightarrow R$	____ , ____
WTS:	$R \vee E$	
3	P	____ , ____
4	S	____ , ____
5	R	____ , ____
6	$R \vee E$	____ , ____

Proof without TFL Sentences after Premises.

1	$(Q \wedge P) \rightarrow S$	Pr.
2	$P \wedge Q$	Pr.
WTS:	$S \wedge Q$	
3	________	∧E, 2
4	________	∧E, 2
5	________	∧I, 3, 4
6	________	→E, 1, 5
7	________	∧I, 4, 6

New Rules: Adding "Layers" to our proofs!

The remaining rules in our Natural Deduction system for TFL will involves **Assumptions** and **Sub-proofs**.

These rules will indicate when you are allowed to start a *new* proof **within** your current proof.

The simplest form of this comes from Conditional Introduction ("$\rightarrow$I").

Rule: Conditional Introduction (“→I” or “→In”)

Conditional Introduction is a rule that lets you write down a **conditional** statement (i.e. a sentence whose main connective is ‘→’), given some lines earlier in your proof. What would this even look like? What—*IN GENERAL*—is required for “$P \rightarrow Q$” to be true?

Rule: Conditional Introduction ("$\rightarrow$I" or "$\rightarrow$In")

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First Pass: Surely, **IF** "P" is true, **THEN** "Q" must also be true.

Rule: Conditional Introduction ("→I" or "→In")

What—*IN GENERAL*—is required for "$P \to Q$" to be true?

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But it also *doesn't require P to actually be true*! So how could we make sense of this in our proof system?

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But how could we—while in the middle of a proof!—show that some *other* argument is valid??

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But how could we—while in the middle of a proof!—show that some *other* argument is valid??

Simplest Solution: Put a proof on your proof so you can prove while you prove! So how the heck would *that* work?!

Sub-proofs and Assumptions

Within a proof, you can start **another** proof once you've moved past the premises stage:

1		$A \to C$	Pr.

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WTS:	$A \to B$		

WANT TO SHOW

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Any line you add *within* your sub-proof can reference any line above it, ***including lines from your main proof!***

Sub-proofs and Assumptions

1	$A \to C$	Pr.
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WTS:	$A \to B$	
3	A	Ass.
4	C	$\to$E, 1, 3

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4		C	$\to$E, 1, 3
5		B	$\to$E, 2, 4

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6		

But it doesn't work in reverse! You *cannot* reference any line from the sub-proof after you've exited and returned to the main proof!

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6	~~B~~	~~Reit, 5~~

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However, there are some rules that can only be applied *if a sub-proof has occurred earlier in the proof*! One such rule is $\rightarrow$I

Sub-proofs and Assumptions

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WTS:	$A \rightarrow B$	
3	A	Ass.
4	C	$\rightarrow$E, 1, 3
5	B	$\rightarrow$E, 2, 4
6		$\rightarrow$I, 3–5

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Sub-proofs and Assumptions

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2	$C \to B$	Pr.
WTS:	$A \to B$	
3	A	Ass.
4	C	$\to$E, 1, 3
5	B	$\to$E, 2, 4
6	$A \to B$	$\to$I, 3–5

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Conditional Introduction appeals to the following kind of reasoning:

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Conditional Introduction appeals to the following kind of reasoning:

Since $\dfrac{\textbf{1.}\ \ P}{\therefore\ \ Q}$ is a valid argument, it must be true that $P \rightarrow Q$

If we assume that it’ll rain tonight, then it follows that I’ll get my hair wet. So, *no matter what* the weather’s like, we know that the conditional “If it rains, then I’ll get my hair wet” is true.

Rule: Conditional Introduction ("→I" or "→In")

Conditional Introduction, "→I"

⋮	⋮	⋮
n	$\mathscr{A}$	Ass.
⋮	⋮	⋮
m	$\mathscr{B}$	(Some Rule) , (Line Numbers)
⋮	⋮	⋮
v	$\mathscr{A} \rightarrow \mathscr{B}$	→I , n–m

Rule: Disjunction Elimination (“$\vee$E” or “$\vee$Out”)

Disjunction Elimination *also* relies on the use of sub-proofs!

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Disjunction Elimination *also* relies on the use of sub-proofs!

1. X is either in the library or in the student center.

2. If X is in the library, then X is on College Ave.

3. If X is in the student center, then X is on College Ave.

$\therefore$ X is on College Ave!

Rule: Disjunction Elimination ("∨E" or "∨Out")

Disjunction Elimination, "∨E"

i	$\mathscr{A} \vee \mathscr{B}$	(Some Rule) , (Line Numbers)
⋮	⋮	⋮
n	$\mathscr{A}$	Ass.
m	$\mathscr{C}$	(Some Rule) , (Line Numbers)
⋮		⋮
u	$\mathscr{B}$	Ass.
v	$\mathscr{C}$	(Some Rule) , (Line Numbers)
j	$\mathscr{C}$	∨E , i, n–m, u–v

Rule: Negation Elimination ("¬E" or "XXIn" or "⊥In")

Negation Elimination is also sometimes known as Contradiction Introduction, because it only "eliminates" negations in a special case, when its the negation of a sentence that appears elsewhere in your proof.

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Negation Elimination, "¬E"

⋮	⋮	⋮
n	$\mathscr{A}$	(Some Rule) , (Line Numbers)
⋮	⋮	⋮
m	$\neg\mathscr{A}$	(Some Rule) , (Line Numbers)
j	$\bot$	¬E , n, m

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n	$\mathscr{A}$	(Some Rule)	(Line Numbers)
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n	$\mathscr{A}$	(Some Rule) , (Line Numbers)	
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Rule: Negation Introduction ("$\neg$I" or "$\neg$In")

Negation introduction is a way of introducing a negation by "eliminating" a contradiction (when that contradiction arises **IN A SUB-PROOF**!). Specifically, if a sub-proof leads you to a contradiction, then you ***know*** that the assumptions you made *cannot* be true!

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Negation Introduction, "¬I"

⋮	⋮	⋮
n	$\mathscr{A}$	Ass.
⋮	⋮	⋮
m	$\bot$	(Some Rule) , (Line Numbers)
j	$\neg\mathscr{A}$	¬I , n–m

Rule: Indirect Proof ("IP")

Indirect Proof, also called "Proof by Contradiction", is extremely similar to negation introduction. However, instead of *introducing* a negation at the end, we're *assuming* a negated statement, in order to show that this leads to a contradiction. If assuming that "P" is false leads to a contradiction, then it must* be that P is true!

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Indirect Proof, "IP"

⋮	⋮	⋮
n	$\quad\neg\mathscr{A}$	Ass.
⋮	$\quad$⋮	⋮
m	$\quad\bot$	(Some Rule) , (Line Numbers)
j	$\mathscr{A}$	IP , n–m

Rule: Explosion (“Ex” or “⊥E” or “XOut”)

The Explosion rule has a fun name because it describes a really wild feature of TFL: Literally anything follows from a contradiction. That is, any argument with a contradiction among its premises is valid, no matter what else is in the argument!

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Explosion, “Ex”

n	$\bot$	(Some Rule) , (Line Numbers)	
$\vdots$	$\vdots$	$\vdots$	
m	$\mathscr{A}$	Ex	, n

1. $\bot$

$\therefore P$

Homework!

From Chapter 15

Block A: Correct both Proofs

Block B: Complete both Proofs

Block C: Do ONLY 1, 2, 3, 4