Class 15 - All of First-Order Logic
Names, Predicates, Quantifiers, and Domains

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Let's do some logic!

The Building Blocks of a Sentence in TFL

Atomic Sentences

Meaning: Stand for whole sentences, but carry no meaning beyond their truth value.

Symbol: Represented by upper-case italicized letters: 'A', 'C', 'X', 'Z', etc.

Sentential Connectives

Meaning: Don't mean anything on their own, but when connected to one or more sentences, they produce a new sentence with a truth-value that is determined by the connective and the truth-values of the parts.

Symbol: Each of the five connectives gets its own symbol: '$\neg$', '$\wedge$', '$\vee$', '$\rightarrow$', '$\leftrightarrow$'.

The Building Blocks of a Sentence in FOL

We'll keep using **Sentential Connectives**, but there are ***NO*** "atomic sentences" in First-Order Logic.

Rather, sentences are made out of the following building blocks:

- Names (aka '*singular terms*')
- Predicates
- Variables
- Quantifiers

Connectives

The Building Blocks of a Sentence in FOL

We'll keep using **Sentential Connectives**, but there are ***NO*** "atomic sentences" in First-Order Logic.

Rather, sentences are made out of the following building blocks:

Names (aka '*singular terms*')

Meaning: Refers to *singular* entities (persons, places, things, time periods, areas, groups, etc.)

Symbol: Represented by lower-case italicized letters between *a* and *r*: '*a*', or '*m*', or '*q*', etc.

abcdef . . . mnopqr

The Building Blocks of a Sentence in FOL

Predicates

Meaning: Refers to *general* things: Things that *you can say about* a singular term. (properties, categories, relationships, verbs, adjectives, etc.). All predicates contain at least one "blank", which can be filled with a name to make a sentence.

Symbol: Represented by upper-case italicized letters, together with a list of VARIABLES, in parentheses, each one representing one "blank": '$F(x)$', '$G(x)$', '$R(x,y)$', '$H(x,y,z)$', '$Z(x,y,z,w)$'.

Variables

Meaning: Don't mean anything on their own. When talking about a predicate *in general*, you represent its empty blanks with variables.

Symbol: Represented by lower-case italicized letters between s and z:

The Building Blocks of a Sentence in FOL

Quantifiers

Meaning:

Symbol:

The Building Blocks of a Sentence in FOL

Quantifiers

Meaning:

Symbol:

This is our topic for today!

Sun is a brighter light than Laptop

Sentences with no names

Most simple sentences can be expressed by following this rule of thumb:

RULE OF THUMB: To make a sentence in FOL, just pick a predicate (like '$R(x, y)$'), and then fill each of its blank(s) with one or more name(s) (like 'b' and 'k'), resulting in a sentence (like '$R(b, k)$').

This “fill-in-the-blanks” rule of thumb works really well for lots of sentences, like:

Catt Damon is a cat.

Professor Plum killed Mr. Body in the Conservatory with the Candlestick.

Beyoncé is taller than Danny Devito

This "fill-in-the-blanks" rule of thumb works really well for lots of sentences, like:

Catt Damon is a cat.

FOL Trans.: '$C(c)$'

Symbol Key: c=Catthew, $C(x) =$ ___x is a cat

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Professor Plum killed Mr. Body in the Conservatory with the Candlestick.

FOL Trans.: '$K(p, b, n, d)$'

Symbol Key: p =Plum, b =Body, n =Conservatory, d =Candlestick, $K(x, y, z, w)$ = $\_\__x$ killed $\_\__y$ in $\_\__z$ with $\_\__w$.

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Beyoncé is taller than Danny Devito

FOL Trans.: '$T(e,o)$'

Symbol Key: e =Beyoncé, o =Devito, $T(x,y) = \_\__x$ is taller than $\_\__y$

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Symbol Key: b =Body, d =Candlestick, $P(x, y)$ =Professor Plum killed $\_\__x$ in the Conservatory with $\_\__y$

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Sentences with empty blanks

But the “fill-in-the-blanks” rule of thumb FAILS for sentences with not enough names in them!

I hear something.

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FOL Trans.: ???

Symbol Key: p=Zee, $H(x,y)$=$\_\__x$ hears $\_\__y$.

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Symbol Key: p=Zee, $H(x,y)$=$\_\__x$ hears $\_\__y$.

Mr. Body was murdered, and the murder weapon was shiny.

FOL Trans.: $M(b) \wedge \ldots$???

Symbol Key: b =Body, $W(x,y) = \_\__x$ was murdered with $\_\__y$,

$M(x) = \_\__x$ was murdered.

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Nobody can run faster than Usain Bolt

FOL Trans.: ???

Symbol Key: r =Bolt $F(x,y) =\_\__x$ can run faster than $\_\__y$

How to make sentences without names

The two kinds of **Quantified** sentences ($A(x) = \_\__x$ is awesome):

Existential Generalization: "Something is awesome"

Universal Generalization: "Everything is awesome"

How to make sentences without names

The two kinds of **Quantified** sentences ($A(x) =$ ___$_x$ is awesome):

EXISTENTIAL GENERALIZATION: "Something is awesome"

UNIVERSAL GENERALIZATION: "Everything is awesome"

Existential Generalization

* We wanna say *something or other* is filling the blank in the predicate "___$_x$ is awesome" WITHOUT having to name it.
* Let's introduce a symbol: '∃', the "Existential Quantifier"
* We'll use '∃' to say "something occupies the blank in '$A(x)$',"
* We use the variable to pick which blank we're talking about, like this: "$\exists x(A(x))$" = "Something is awesome"

How to make sentences without names

The two kinds of **Quantified** sentences ($A(x) = \_\__x$ is awesome):

Existential Generalization: "Something is awesome"

Universal Generalization: "Everything is awesome"

Existential Generalization

* We wanna say there **EXISTS** some name which, if filled into the blank marked 'x' in "$\_\__x$ is awesome", produces a true sentence.
* Let's introduce a symbol: '$\exists$', the "Existential Quantifier"
* We'll use '$\exists$' to say "something occupies the blank in '$A(x)$',"
* We use the variable to pick which blank we're talking about, like this:
  "$\exists x(A(x))$" = "Something is awesome"

How to make sentences without names

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The two kinds of **Quantified** sentences ($A(x) = \_\__x$ is awesome):

EXISTENTIAL GENERALIZATION: "Something is awesome"

UNIVERSAL GENERALIZATION: "Everything is awesome"

Universal Generalization

* We wanna say *every single thing* fills the blank in "$\_\__x$ is awesome" WITHOUT having to list them all.
* Let's introduce a symbol: '∀', the "Universal Quantifier"
* We'll use '∀' to say "everything occupies the blank in '$A(x)$',"
* We use the variable to pick which blank we're talking about, like this: "$\forall x(A(x))$" = "Everything is awesome"

How to make sentences without names

The two kinds of **Quantified** sentences ($A(x) = \_\__x$ is awesome):

Existential Generalization: "Something is awesome"

Universal Generalization: "Everything is awesome"

Universal Generalization

* We wanna say for **ALL** names, you could fill into the blank marked 'x' in "$\_\__x$ is awesome" and always get a true sentence.
* Let's introduce a symbol: '$\forall$', the "Universal Quantifier"
* We'll use '$\forall$' to say "everything occupies the blank in '$A(x)$',"
* We use the variable to pick which blank we're talking about, like this:
  "$\forall x(A(x))$" = "Everything is awesome"

Returning to our rule of thumb

Here's how our rule of thumb was limited:

Rule of Thumb: To make a sentence in FOL, just pick a predicate (like '$R(x, y)$'), and then fill each of its blank(s) with one or more name(s) (like 'b' and 'k'), resulting in a sentence (like '$R(b, k)$').

It works for sentences without quantifiers, but you could also use a quantifier to make a sentence *without* filling in names in all the blanks! Rather, quantifiers let you make sentences by talking about the blanks (the variables) *themselves*.

Returning to our rule of thumb

Here's how our rule of thumb was limited:

Rule of Thumb: To make a n *UN*-quantified sentence in FOL, just pick a predicate (like '$R(x,y)$'), and then fill each of its blank(s) with one or more name(s) (like 'b' and 'k'), resulting in a sentence (like '$R(b,k)$').

It works for sentences without quantifiers, but you could also use a quantifier to make a sentence *without* filling in names in all the blanks! Rather, quantifiers let you make sentences by talking about the blanks (the variables) *themselves*.

Sentences with Quantifiers

Let's see how this works for these sentences:

I hear something.

Sentences with Quantifiers

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$H(p, \ )$

Symbol Key: p=Zee, $H(x, y)$=__$_x$ hears __$_y$.

Sentences with Quantifiers

Let's see how this works for these sentences:

I hear something.

FOL Trans.: $\exists y(H(p,y))$

Symbol Key: p=Zee, $H(x,y)$=$\_\__x$ hears $\_\__y$.

$\exists y(Hpy)$

Sentences with Quantifiers

Let's see how this works for these sentences:

I hear something.

FOL Trans.: $\exists y(H(p,y))$

Symbol Key: p=Zee, $H(x,y)$=$\_\__x$ hears $\_\__y$.

Nobody can run faster than Usain Bolt

Symbol Key: r =Bolt $F(x,y)$ =$\_\__x$ can run faster than $\_\__y$

$\exists y(F(r,y))$

Sentences with Quantifiers

Let's see how this works for these sentences:

I hear something.

FOL Trans.: $\exists y(H(p,y))$

Symbol Key: p=Zee, $H(x,y)$=$\_\__x$ hears $\_\__y$.

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FOL Trans.: $\forall x(\neg F(x,r))$,

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Mr. Body was murdered, and the murder weapon was shiny.

FOL Trans.: $M(b) \wedge$

Symbol Key: b =Body, $W(x, y) = \_\__x$ was murdered with $\_\__y$,
$M(x) = \_\__x$ was murdered, $S(x) = \_\__x$ was shiny

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FOL Trans.: $\forall x(\neg F(x, r))$,

Symbol Key: r =Bolt $F(x, y) = \_\__x$ can run faster than $\_\__y$

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FOL Trans.: $M(b) \wedge \exists x(W(b,x) \wedge S(x))$

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FOL Trans.: $\forall x(\neg F(x,r))$,

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Sentences with Quantifiers

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Symbol Key: p=Zee, $H(x,y)$=$\_\__x$ hears $\_\__y$.

Mr. Body was murdered, and the murder weapon was shiny.

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Symbol Key: b =Body, $W(x,y) =\_\__x$ was murdered with $\_\__y$, $M(x) =\_\__x$ was murdered, $S(x) =\_\__x$ was shiny

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FOL Trans.: $\forall x(\neg F(x,r))$, OR $\neg\exists x(F(x,r))$

Symbol Key: r =Bolt $F(x,y) =\_\__x$ can run faster than $\_\__y$

Two ways to make a sentence

There are two ways to make a sentence in FOL:

1. Fill-in every variable (blank) in your predicate(s) with a name.
2. Leave some of your variables (blanks) un-filled, but ***BIND*** them with a quantifier out in front of the sentence.

Two ways to make a sentence

There are two ways to make a sentence in FOL:

1. Fill-in every variable (blank) in your predicate(s) with a name.
2. Leave some of your variables (blanks) un-filled, but ***BIND*** them with a quantifier out in front of the sentence.

If you write down a predicate and don't fill in all the blanks, the variables labeling those blanks are called "unbound" variables. You can either remove those variables by filling in their blanks or by using a quantifier on the sentence that refers to those blanks using that variable.

https://tinyurl.com/AttendQuizOct27

Remember our original arguments?

1. Sirina McKitten is a dog.
2. All dogs are adorable.

C. Sirina McKitten is adorable.

We can now translate every part of it!

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Quizlet Q1: Figure out the names and predicates, and write down a translation key.

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Quizlet Q1: Figure out the names and predicates, and write down a translation key.

$k =$Sirina McKitten, $D(x) = \_\__x$ is a dog, $A(x) = \_\__x$ is adorable.

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C. Sirina McKitten is a adorable.

We can now translate every part of it!

$k =$Sirina McKitten, $D(x) = \_\__x$ is a dog, $A(x) = \_\__x$ is adorable.

Quizlet Q2: Using this translation key, translate this argument.

https://tinyurl.com/AttendQuizOct27

Remember our original arguments?

1. $D(k)$
2. All dogs are adorable.
C. Sirina McKitten is a adorable.

We can now translate every part of it!

$k =$Sirina McKitten, $D(x) = \_\__x$ is a dog, $A(x) = \_\__x$ is adorable.

https://tinyurl.com/AttendQuizOct27

Remember our original arguments?

No! Bad!

~~$\forall x(Dx \wedge Ax)$~~

1. $D(k)$
2. All dogs are adorable.

C. $A(k)$

We can now translate every part of it!

$k =$Sirina McKitten, $D(x) =$ __$_x$ is a dog, $A(x) =$ __$_x$ is adorable.

https://tinyurl.com/AttendQuizOct27

Remember our original arguments?

1. $D(k)$
2. $\forall x(\ D(x) \to A(x)\ )$

C. $A(k)$

We can now translate every part of it!

$k =$Sirina McKitten, $D(x) = \_\__x$ is a dog, $A(x) = \_\__x$ is adorable.

https://tinyurl.com/AttendQuizOct27

Remember our original arguments?

1. $D(k)$
2. $\forall x(\ D(x) \rightarrow A(x)\ )$

C. $A(k)$

Premise 2, literally, reads "Everything is such that, if it's a dog, then it's adorable", or "For all x, if x is a dog, then x is adorable".

B:	→	G
T	T	T
F	T	T
T	F	F
F	T	F

We end up needing another thing in our symbolization key, a **domain**.

1. $D(k)$
2. $\forall x(\ D(x) \rightarrow A(x)\ )$

C. $A(k)$

A domain determines what the quantifiers "range over", i.e. they determine which things are counted as part of the "all", or which ones matter when determining if "something" really does occupy the blank.

Domain: animals on Earth
$k =$Sirina McKitten, $D(x) = \_\__x$ is a dog, $A(x) = \_\__x$ is adorable.

We end up needing another thing in our symbolization key, a **domain**

1. $D(k)$
2. $\forall x(\ D(x) \rightarrow A(x)\ )$

C. $A(k)$

A domain determines what the quantifiers "range over", i.e. they determine which things are counted as part of the "all", or which ones matter when determining if "something" really does occupy the blank.

Domain: creatures in the universe

$k =$Sirina McKitten, $D(x) = \_\__x$ is a dog, $A(x) = \_\__x$ is adorable.

We end up needing another thing in our symbolization key, a **domain**

1. $D(k)$
2. $\forall x(\ D(x) \rightarrow A(x)\ )$

C. $A(k)$

A domain determines what the quantifiers "range over", i.e. they determine which things are counted as part of the "all", or which ones matter when determining if "something" really does occupy the blank.

Domain: ~~mammals in Philade~~lphia
$k =$Sirina McKitten, $D(x) = \_\__x$ is a dog, $A(x) = \_\__x$ is adorable.

We end up needing another thing in our symbolization key, a **domain**

1. $D(k)$
2. $\forall x(\ D(x) \rightarrow A(x)\ )$

C. $A(k)$

A domain determines what the quantifiers "range over", i.e. they determine which things are counted as part of the "all", or which ones matter when determining if "something" really does occupy the blank.

Domain: ~~creatures in Philadelphia~~
$k =$Sirina McKitten, $D(x) = \_\__x$ is a dog, $A(x) = \_\__x$ is adorable.

Homework

Read: Chapters 21, 22, and 23

RE-Do: Homework 2, ***but using FOL instead of TFL!*** (Chapter 5 Question Block C, question 2 plus all the odd questions, Question Block E, all questions, and Question Block H question 1)

Do: Chap 22: Problem Block A, Questions 1, 4, 8, 12, 14

Do: Chap 22, Problem Block B, all of it

Do: Chap 22, Problem Block D: ONLY do Question 1, and Question 3

Chap 23!