

Class 10 Proof Strategies in TFL

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Let's do some logic

Diagnosing an Incorrect "Proof"

1	$(\neg L \wedge A) \vee L$	Pr.
2	$\neg L \wedge A$	Ass.
3	$\neg L$	$\wedge E, 2$
4	A	$\wedge E, 2$
5	L	Ass.
6	\perp	$\neg E, 3, 5$
7	A	$Ex, 6$
8	A	$\vee E, 1, 2-4, 5-7$

$\perp = \text{Contradiction}$

First
Incorrect
"Proof":

Diagnosing an Incorrect "Proof"

Second
Incorrect
"Proof":

1	$A \wedge (B \wedge C)$	Pr.
2	$(B \vee C) \rightarrow D$	Pr.
3	B	$\wedge E, 1$
4	$B \vee C$	$\vee I, 3$
5	D	$\rightarrow E, 4, 2$ ✓

Proof without Rules & Line Numbers

1	$P \wedge S$	<u>Pr</u> , <u> </u>
2	$S \rightarrow R$	<u>Pr</u> , <u> </u>
3	P	<u>$\wedge E$</u> , <u>1</u>
4	S	<u>$\wedge E$</u> , <u>1</u>
5	R	<u>$\rightarrow E$</u> , <u>2,4</u>
6	$R \vee E$	<u>$\vee I$</u> , <u>5</u>

1	$A \rightarrow D$	<u>Pr</u> , <u> </u>
2	$A \wedge B$	<u>Ass</u> , <u> </u>
3	A	<u>$\wedge E$</u> , <u>2</u>
4	D	<u>$\rightarrow E$</u> , <u>1,3</u>
5	$D \vee E$	<u>$\vee I$</u> , <u>4</u>
6	<u>$(A \wedge B) \rightarrow (D \vee E)$</u>	<u>$\rightarrow I$</u> , <u>2-5</u>

Proof without Rules & Line Numbers

1		$\neg L \rightarrow (J \vee L)$	_____ , _____
2		$\neg L$	_____ , _____
<hr/>			
3		$J \vee L$	_____ , _____
4			
4		J	_____ , _____
<hr/>			
5		$J \wedge J$	_____ , _____

6		J	_____ , _____
7		L	_____ , _____
<hr/>			
8		\perp	_____ , _____
9		J	_____ , _____
<hr/>			
10		J	_____ , _____

Rule: Reiteration (“Reit.” or “R”)

The idea is this: The following argument-form is always valid, no matter what sentence you use.

$$\frac{1. P}{\therefore P}$$

$$\frac{1. (A \leftrightarrow G)}{\therefore (A \leftrightarrow G)}$$

$$\frac{1. \text{I'm busy}}{\therefore \text{I'm busy}}$$

Reiteration Rule, “Reit.”

	⋮	
n		\mathcal{A} (SOME RULE), (LINE NUMBERS)
⋮		⋮
m		\mathcal{A} Reit., n

Rule: Conjunction Introduction (“ $\wedge I$ ” or “ $\wedge In$ ”)

$\frac{1. P \quad 2. Q \vee P}{\therefore P \wedge (Q \vee P)}$	$\frac{1. (A \leftrightarrow G) \quad 2. \neg S}{\therefore (A \leftrightarrow G) \wedge \neg S}$	$\frac{1. \text{There's a giraffe in this building} \quad 2. \text{NYU Shanghai is in China}}{\therefore \text{There's a giraffe in this building and NYU Shanghai is in China}}$
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Conjunction Introduction, “ $\wedge I$ ”

n	\mathcal{A}				(SOME RULE) , (LINE NUMBERS)
:	:				
m	\mathcal{B}				(SOME RULE) , (LINE NUMBERS)
:	:				
u	$\mathcal{A} \wedge \mathcal{B}$		$\wedge I$,	n, m

Rule: Conjunction Elimination (“ $\wedge E$ ” or “ $\wedge Out$ ”)

$$\frac{1. P \wedge (Q \vee P)}{\therefore Q \vee P}$$

$$\frac{1. \neg S \wedge (A \leftrightarrow G)}{\therefore \neg S}$$

1. There's a giraffe in this building
and NYU Shanghai is in China

 \therefore There's a giraffe in this building

Conjunction Elimination, “ $\wedge E$ ”

n	$A \wedge B$	(SOME RULE), (LINE NUMBERS)
:	:	:
u	A	$\wedge E$, n
:	:	:
v	B	$\wedge E$, n

Rule: Disjunction Introduction (“ $\vee I$ ” or “ $\vee In$ ”)

1. It's Wednesday.

\therefore Either it's Wednesday or
I'm a kettle of fish.

$$\frac{1. \neg S \wedge A}{\therefore (\neg S \wedge A) \vee B}$$

$$\frac{1. P}{\therefore P \vee Q}$$

Disjunction Introduction, “ $\vee I$ ”

For ANY “ B ”!

n		A	(SOME RULE), (LINE NUMBERS)
:		:	:
u		$A \vee B$	$\vee I$, n

Rule: Conditional Elimination (“ \rightarrow E” or “ \rightarrow Out”)

1. If it's raining then my hair
will get wet.

2. It's raining.

\therefore My hair will get wet

1. $(\neg S \wedge A) \rightarrow (B \leftrightarrow A)$

2. $\neg S \wedge A$

$\therefore B \leftrightarrow A$

1. $P \rightarrow Q$

2. P

$\therefore Q$

Conditional Elimination, “ \rightarrow E”

n	$A \rightarrow B$	(SOME RULE), (LINE NUMBERS)
---	-------------------	-----------------------------

:	:	:
---	---	---

m	A	(SOME RULE), (LINE NUMBERS)
---	-----	-----------------------------

:	:	:
---	---	---

u	B	\rightarrow E, n, m
---	---------------------------	-----------------------

Rule: Biconditional Elimination (“ \leftrightarrow E” or “ \leftrightarrow Out”)

1. I go jogging at 6am if and only if someone else runs with me.

2. I go jogging at 6am.

\therefore Someone runs with me.

$$1. (\neg S \wedge A) \leftrightarrow (B \rightarrow A)$$

$$2. \neg S \wedge A$$

$$\therefore B \rightarrow A$$

$$1. P \leftrightarrow Q$$

$$2. P$$

$$\therefore Q$$

Biconditional Elimination, “ \leftrightarrow E”

n $A \leftrightarrow B$ (SOME RULE), (LINE NUMBERS)

∴ ∴ ∴

m A (SOME RULE), (LINE NUMBERS)

∴ ∴ ∴

u B \leftrightarrow E, n,m

Rule: Biconditional Elimination (“ $\leftrightarrow E$ ” or “ $\leftrightarrow Out$ ”)

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2. Someone runs with me.

\therefore I go jogging at 6am.

$$1. (\neg S \wedge A) \leftrightarrow (B \rightarrow A)$$

$$2. B \rightarrow A$$

$$\therefore \neg S \wedge A$$

$$1. P \leftrightarrow Q$$

$$2. Q$$

$$\therefore P$$

Biconditional Elimination, “ $\leftrightarrow E$ ”

n $A \leftrightarrow B$ (SOME RULE), (LINE NUMBERS)

∴ ∴ ∴

m B (SOME RULE), (LINE NUMBERS)

∴ ∴ ∴

u A $\leftrightarrow E$, n,m

Rules that involve SUB-PROOFS

There are two kinds of rules in Natural Deduction for TFL:

Rules that care about
previous lines in the *main*
proof

- Reit
- $\wedge E$
- $\wedge I$
- $\vee I$
- $\rightarrow E$
- $\leftrightarrow E$

Rules that *also* care about
previous *sub-proofs*

- $\rightarrow I$
- $\leftrightarrow I$
- $\vee E$
- $\neg I$
- $\neg E$
- IP
- Ex

What is a sub-proof? Roughly: You are allowed to start a *new* proof **within** your current proof, and these rules will depend on there being

Rules that involve SUB-PROOFS

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Quizlet! Q1: Which of these rules do you find most confusing/do you want more examples of?

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Rule: Conditional Introduction (“ \rightarrow I” or “ \rightarrow In”)

Conditional Introduction is a rule that lets you write down a **conditional** statement (i.e. a sentence whose main connective is ‘ \rightarrow ’), given some lines earlier in your proof. What would this even look like? What—*IN GENERAL*—is required for “ $P \rightarrow Q$ ” to be true?

Rule: Conditional Introduction (“ \rightarrow I” or “ \rightarrow In”)

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First Pass: Surely, IF “ P ” is true, THEN “ Q ” must also be true.

Rule: Conditional Introduction (“ \rightarrow I” or “ \rightarrow In”)

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But “ $P \rightarrow Q$ ” being true *doesn't require* P to actually be true! So how could we make sense of this in our proof system?

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Observation: Actually, this is kind of familiar... You know what's also like that? **VALIDITY!**

Second Pass: So “ $P \rightarrow Q$ ” is true whenever $\frac{1. P}{\therefore Q}$ is valid.

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Solution: Start a “sub-proof” by making a **temporary** assumption, and considering what follows from it!

Sub-proofs and Assumptions

Within a proof, you can start another proof once you've moved past the premises stage:

1 | $A \rightarrow C$ Pr.

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Sub-proofs and Assumptions

When you start a sub-proof, you introduce *Assumptions* instead of premises.

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<hr/>			
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3			

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<hr/>			
WTS:		$A \rightarrow B$	
3			Ass.
		A	
4			

Sub-proofs and Assumptions

1 $A \rightarrow C$ Pr.
2 $C \rightarrow B$ Pr.

WTS: $A \rightarrow B$

3 A Ass.
4 C $\rightarrow E, 1, 3$

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4 C $\rightarrow E, 1, 3$
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3			A	Ass.
4			C	$\rightarrow E, 1, 3$
5			B	$\rightarrow E, 2, 4$
6				

But it doesn't work in reverse! You *cannot* reference any line from the sub-proof after you've exited and returned to the main proof!

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5			B	$\rightarrow E, 2, 4$
6			B	Reit, 5

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6				

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However, there are some rules that can only be applied *if a sub-proof has occurred earlier in the proof!*

One such rule is $\rightarrow I$

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<hr/>			
WTS:		$A \rightarrow B$	
3			
		A	Ass.
<hr/>			
4		C	$\rightarrow E, 1, 3$
5		B	$\rightarrow E, 2, 4$
6			$\rightarrow I, 3-5$

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		A	Ass.
<hr/>			
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5		B	$\rightarrow E, 2, 4$
6		$A \rightarrow B$	$\rightarrow I, 3-5$

However, there are some rules that can only be applied *if a sub-proof has occurred earlier in the proof!*

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“3–5” refers to the *whole sub-proof!*

Rule: Conditional Introduction (“ $\rightarrow I$ ” or “ $\rightarrow In$ ”)

Conditional Introduction appeals to the following kind of reasoning:

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If we **ASSUME** it'll rain tonight, then it follows that I get my hair wet.
So, *no matter what the weather's like*, we know the conditional is true.

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Conditional Introduction, " $\rightarrow I$ "

\vdots	\vdots	\vdots	
n	A	Ass.	
\vdots	\vdots	\vdots	
m	B	(SOME RULE), (LINE NUMBERS)	
\vdots	\vdots	\vdots	
u	$A \rightarrow B$	$\rightarrow I$, n-m

Rule: Disjunction Elimination (“ $\vee E$ ” or “ $\vee Out$ ”)

Disjunction Elimination *also* relies on the use of sub-proofs!

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Disjunction Elimination *also* relies on the use of sub-proofs!

1. X is either in the B1 Cafeteria or in the 2nd-floor Cafe.
2. If X is in the B1 Cafeteria, then X is in 1555 Century Ave .

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1. X is either in the B1 Cafeteria or in the 2nd-floor Cafe.
2. If X is in the B1 Cafeteria, then X is in 1555 Century Ave .
3. If X is in the 2nd-floor Cafe, then X is in 1555 Century Ave.

$\therefore X$ is in 1555 Century Ave!

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Disjunction Elimination, “ $\vee E$ ”

i	$A \vee B$	(SOME RULE) , (LINE NUMBERS)
:	:	:
n	A	Ass.
m	C	(SOME RULE) , (LINE NUMBERS)
:	:	:
u	B	Ass.
v	C	(SOME RULE) , (LINE NUMBERS)
j	C	$\vee E$, i, n-m, u-v

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n	A	Ass.
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:	:	:
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v	C	(SOME RULE) , (LINE NUMBERS)
j	C	$\vee E$, i, n-m, u-v

Rule: Negation Elimination (“ $\neg E$ ” or “ ~~$\exists I$~~ ” or “ $\perp I$ ”)

Negation Elimination is also sometimes known as Contradiction Introduction, because it only “eliminates” negations in a special case, when its the negation of a sentence that appears elsewhere in your proof.

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\vdots	\vdots	\vdots
n	\mathcal{A}	(SOME RULE) , (LINE NUMBERS)
\vdots	\vdots	\vdots
m	$\neg \mathcal{A}$	(SOME RULE) , (LINE NUMBERS)
j	\perp	$\neg E$, n, m

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:	:	:
n	\cancel{X}	(SOME RULE) , (LINE NUMBERS)
:	:	:
m	$\neg \cancel{X}$	(SOME RULE) , (LINE NUMBERS)
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\vdots	\vdots	\vdots
m	$\neg \mathcal{A}$	(SOME RULE) , (LINE NUMBERS)
j	\perp	$\neg E$, n, m

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Negation introduction is a way of introducing a negation by “eliminating” a contradiction (when that contradiction arises **IN A SUB-PROOF!**). Specifically, if a sub-proof leads you to a contradiction, then you *know* that the assumptions you made *cannot* be true!

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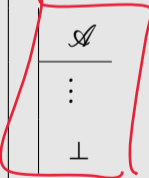

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Negation Introduction, “ $\neg I$ ”

\vdots	\vdots	\vdots
n	 Ass.	
\vdots	\vdots	\vdots
m	\perp	(SOME RULE), (LINE NUMBERS)
j	\neg 	$\neg I$, n-m

Rule: Indirect Proof (“IP”)

Indirect Proof, also called “Proof by Contradiction”, is extremely similar to negation introduction. However, instead of *introducing* a negation at the end, we’re *assuming* a negated statement, in order to show that this leads to a contradiction. If assuming that “ P ” is false leads to a contradiction, then it must* be that P is true!

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Indirect Proof, “IP”

⋮	⋮	⋮
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⋮	⋮	⋮
m	\perp	(SOME RULE) , (LINE NUMBERS)
j	\mathcal{A}	IP , n-m

Rule: Explosion (“Ex” or “ \perp E” or ‘XOut’)

The Explosion rule has a fun name because it describes a really wild feature of TFL: Literally anything follows from a contradiction. That is, any argument with a contradiction among its premises is valid, no matter what else is in the argument!

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Explosion, “Ex”

n	\perp	(SOME RULE) , (LINE NUMBERS)
:	:	:
m	\mathcal{A}	Ex , n

The rest of the Homework!

Chapter 15 Block C

Proofs 1, 2, 3, and 4.

1. $J \rightarrow \neg J \therefore \neg J$

2. $Q \rightarrow (Q \wedge \neg Q) \therefore \neg Q$

3. $A \rightarrow (B \rightarrow C) \therefore (A \wedge B) \rightarrow C$

4. $K \wedge L \therefore K \leftrightarrow L$

How to Construct Good Proofs!

Work backwards from what you want

- * More often than not, there are only a few rules that will be able to output a sentence like your desired conclusion.
- * If you arrived at your concluding line using one of those rules, what other lines would the **rest** of your proof have to contain?

Work forwards from what you have

- * If you've got some starting sentences, there are usually a few lines that you can do just by figuring out what you can extract from them.
- * That is, use any elimination rules that are appropriate to the main connective of your premises, and "pull out" whatever you can into its own line.

How to Construct Good Proofs!

Give yourself mini-goals (but don't lose sleep if you miss 'em)

- * When working backwards from your desired conclusion, make your mini-goal those sentences required to get to that point.
- * When working forwards from your existing premises/lines, you might find a rule that you cannot use to work forward, because it requires a line you don't have. Make your mini-goal the lines that will allow you to use more of those rules.

Having trouble? Try Proof by Contradiction ("IP")

- * If you're not sure of how to proceed, whether because you ran out of ways to work forward/back or because you're stumped, try this:
- * Start a sub-proof, and assume **the negation** of your conclusion! Then, saying within that sub-proof, try to prove a contradiction (" \perp ").

Proof Practice!

Let's do some proofs together!

$$1 \quad \neg(P \wedge \neg P)$$

$$2 \quad \neg\neg A \rightarrow A$$

$$3 \quad \neg A \rightarrow (A \rightarrow \perp)$$

$$4 \quad (A \vee \neg B) \rightarrow (B \rightarrow A)$$

$$1. \quad | \quad \neg A$$

Ass.

$$2. \quad | \quad | \quad A$$

Ass.

$$3. \quad | \quad | \quad \perp$$

$$4. \quad | \quad A \rightarrow \perp$$

$$5. \quad | \quad \neg A \rightarrow (A \rightarrow \perp)$$

~~$\neg E$~~ I, 1, 2

$\rightarrow I$, 2-3

$\rightarrow I$, 1-4