Class 11 Proof Strategies and Proof-Theoretic Concepts

Zee R. Perry

Let's do some logic

Diagnosing an Incorrect "Proof"

1	$(\neg L \wedge A) \vee L$	Pr.
2	$\neg L \wedge A$	Ass.
3	$\neg L$	$\wedge$E, 3
4	A	$\wedge$E, 1
5	L	Ass.
6	$\bot$	$\neg$E, 3,5
7	A	Ex, 6
8	A	$\vee$E, 1, 2–4, 5–7

$\bot$ = Contradiction

First Incorrect "Proof":

Diagnosing an Incorrect "Proof"

Second Incorrect "Proof":

1	$A \wedge (B \wedge C)$	Pr.
2	$(B \vee C) \rightarrow D$	Pr.
3	B	$\wedge$E, 1
4	$B \vee C$	$\vee$I, 3
5	D	$\rightarrow$E, 4, 2

Proof without Rules & Line Numbers

1	$P \wedge S$	Pr	
2	$S \rightarrow R$	Pr	
3	P	$\wedge$E	1
4	S	$\wedge$E	1
5	R	$\rightarrow$E	2,4
6	$R \vee E$	$\vee$I	5

1	$A \rightarrow D$	Pr	
2	$A \wedge B$	Ass	
3	A	$\wedge$E	2
4	D	$\rightarrow$E	1,3
5	$D \vee E$	$\vee$I	4
6	$(A \wedge B) \rightarrow (D \vee E)$	$\rightarrow$I	2-5

Proof without Rules & Line Numbers

10 | J

∨E = J ∨ L (3) + [J … J] (4–6) + [L … J] (7–9)

Line	Formula	Rule	Lines
1	$\neg L \to (J \lor L)$	Pr	
2	$\neg L$	Pr	
3	$J \lor L$	→E	1, 2
4	$\quad J$	Ass	
5	$\quad J \land J$	∧I	4, 4
6	$\quad J$	∧E	5
7	$\quad L$	Ass	
8	$\quad \bot$	¬E	2, 7
9	$\quad J$	Ex	8
10	J	∨E	3, 4–6, 7–9

Rules that involve SUB-PROOFS

There are two kinds of rules in Natural Deduction for TFL:

Rules that care about previous lines in the *main* proof

- Reit
- $\wedge$E
- $\wedge$I
- $\vee$I
- $\rightarrow$E
- $\leftrightarrow$E

Rules that *also* care about previous *sub-proofs*

- $\rightarrow$I
- $\leftrightarrow$I
- $\vee$E
- $\neg$I
- $\neg$E
- IP
- Ex

What is a sub-proof? Roughly: You are allowed to start a *new* proof **within** your current proof, and these rules will depend on there being

Rule: Reiteration ("Reit." or "R")

The idea is this: The following argument-form is always valid, no matter what sentence you use.

1. P	**1.** $(A \leftrightarrow G)$	**1.** I'm busy
$\therefore$ P	$\therefore$ $(A \leftrightarrow G)$	$\therefore$ I'm busy

Reiteration Rule, "Reit."

	$\vdots$	
n	$\mathscr{A}$	(Some Rule) , (Line Numbers)
$\vdots$	$\vdots$	$\vdots$
m	$\mathscr{A}$	Reit., n

Rule: Conjunction Introduction ("$\wedge$I" or "$\wedge$In")

1. P
2. $Q \vee P$

$\therefore\ P \wedge (Q \vee P)$

1. $(A \leftrightarrow G)$
2. $\neg S$

$\therefore\ (A \leftrightarrow G) \wedge \neg S$

1. There's a giraffe in this building
2. NYU Shanghai is in China

$\therefore$ There's a giraffe in this building *and* NYU Shanghai is in China

Conjunction Introduction, "$\wedge$I"

n	$\mathscr{A}$	(Some Rule) , (Line Numbers)
$\vdots$	$\vdots$	$\vdots$
m	$\mathscr{B}$	(Some Rule) , (Line Numbers)
$\vdots$	$\vdots$	$\vdots$
u	$\mathscr{A} \wedge \mathscr{B}$	$\wedge$I , n, m

Rule: Conjunction Elimination ("∧E" or "∧Out")

1. $P \wedge (Q \vee P)$

$\therefore\ Q \vee P$

1. $\neg S \wedge (A \leftrightarrow G)$

$\therefore\ \neg S$

1. There's a giraffe in this building *and* NYU Shanghai is in China

∴ There's a giraffe in this building

Conjunction Elimination, "∧E"				
n	$\mathscr{A} \wedge \mathscr{B}$	(Some Rule)	,	(Line Numbers)
⋮	⋮	⋮		
u	$\mathscr{A}$	∧E	,	n
⋮	⋮	⋮		
v	$\mathscr{B}$	∧E	,	n

Rule: Disjunction Introduction ("∨I" or "∨In")

1. It's Wednesday.

∴ Either it's Wednesday or I'm a kettle of fish.

1. $\neg S \wedge A$

∴ $(\neg S \wedge A) \vee B$

1. P

∴ $P \vee Q$

Disjunction Introduction, "∨I"

For **ANY** "$\mathscr{B}$" !

n	$\mathscr{A}$	(Some Rule) , (Line Numbers)
⋮	⋮	⋮
u	$\mathscr{A} \vee \mathscr{B}$	∨I , n

Rule: Conditional Elimination ("→E" or "→Out")

1. If it's raining then my hair will get wet.
2. It's raining.

∴ My hair will get wet

1. $(\neg S \wedge A) \to (B \leftrightarrow A)$
2. $\neg S \wedge A$

$\therefore \quad B \leftrightarrow A$

1. $P \to Q$
2. P

$\therefore \quad Q$

Conditional Elimination, "→E"

n	$\mathscr{A} \to \mathscr{B}$	(Some Rule) , (Line Numbers)
⋮	⋮	⋮
m	$\mathscr{A}$	(Some Rule) , (Line Numbers)
⋮	⋮	⋮
u	$\mathscr{B}$	→E , n,m

Rule: Biconditional Elimination ("↔E" or "↔Out")

1. I go jogging at 6am if and only if someone else runs with me.
2. I go jogging at 6am.

∴ Someone runs with me.

1. $(\neg S \wedge A) \leftrightarrow (B \rightarrow A)$
2. $\neg S \wedge A$

∴ $B \rightarrow A$

1. $P \leftrightarrow Q$
2. P

∴ Q

Biconditional Elimination, "↔E"		
n	$\mathscr{A} \leftrightarrow \mathscr{B}$	(Some Rule) , (Line Numbers)
⋮	⋮	⋮
m	$\mathscr{A}$	(Some Rule) , (Line Numbers)
⋮	⋮	⋮
u	$\mathscr{B}$	↔E , n,m

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∴ I go jogging at 6am.

1. $(\neg S \wedge A) \leftrightarrow (B \rightarrow A)$
2. $B \rightarrow A$

$\therefore\ \neg S \wedge A$

1. $P \leftrightarrow Q$
2. Q

$\therefore\ P$

Biconditional Elimination, "↔E"		
n	$\mathscr{A} \leftrightarrow \mathscr{B}$	(Some Rule) , (Line Numbers)
⋮	⋮	⋮
m	$\mathscr{B}$	(Some Rule) , (Line Numbers)
⋮	⋮	⋮
u	$\mathscr{A}$	↔E , n,m

Rule: Conditional Introduction ("→I" or "→In")

Conditional Introduction is a rule that lets you write down a **conditional** statement (i.e. a sentence whose main connective is '→'), given some lines earlier in your proof. What would this even look like? What—*IN GENERAL*—is required for "$P \rightarrow Q$" to be true?

Rule: Conditional Introduction ("$\rightarrow$I" or "$\rightarrow$In")

What—*IN GENERAL*—is required for "$P \rightarrow Q$" to be true?

First Pass: Surely, **IF** "P" is true, **THEN** "Q" must also be true.
But "$P \rightarrow Q$" being true *doesn't require P to actually be true*! So how could we make sense of this in our proof system?

Observation: Actually, this is kind of familiar... You know what's **also** like that? **VALIDITY!**

Second Pass: So "$P \rightarrow Q$" is true whenever

1. P
$\therefore$ Q

is valid.

But how could we—while in the middle of a proof!—show that some *other* argument is valid??

Solution: Start a "sub-proof" by making a **temporary** assumption, and considering what follows from it!

Sub-proofs and Assumptions

Within a proof, you can start **another** proof once you've moved past the premises stage:

1	$A \rightarrow C$	Pr.

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1	$A \rightarrow C$	Pr.
2	$C \rightarrow B$	Pr.
WTS:	$A \rightarrow B$	

WTS = Want to Show

Sub-proofs and Assumptions

When you start a sub-proof, you introduce ***Assumptions*** instead of premises.

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3		

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4	C	

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Any line you add *within* your sub-proof can reference any line above it, ***including lines from your main proof!***

Sub-proofs and Assumptions

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2	$C \rightarrow B$	Pr.
WTS:	$A \rightarrow B$	
3	A	Ass.
4	C	$\rightarrow$E, 1, 3

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Sub-proofs and Assumptions

1	$A \to C$	Pr.
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4	C	$\to$E, 1, 3
5	B	$\to$E, 2, 4

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Sub-proofs and Assumptions

1	$A \to C$	Pr.
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WTS:	$A \to B$	
3	A	Ass.
4	C	$\to$E, 1, 3
5	B	$\to$E, 2, 4
6		

But it doesn't work in reverse! You *cannot* reference any line from the sub-proof after you've exited and returned to the main proof!

Sub-proofs and Assumptions

1	$A \to C$	Pr.
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WTS:	$A \to B$	
3	A	Ass.
4	C	$\to$E, 1, 3
5	B	$\to$E, 2, 4
6	B	Reit, 5

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2	$C \to B$	Pr.
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4	C	$\to$E, 1, 3
5	B	$\to$E, 2, 4
6	~~B~~	~~Reit, 5~~

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Sub-proofs and Assumptions

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2	$C \rightarrow B$	Pr.
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5	B	$\rightarrow$E, 2, 4
6		

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However, there are some rules that can only be applied *if a sub-proof has occurred earlier in the proof*! One such rule is $\rightarrow$I

Sub-proofs and Assumptions

1	$A \to C$	Pr.
2	$C \to B$	Pr.
WTS:	$A \to B$	
3	A	Ass.
4	C	$\to$E, 1, 3
5	B	$\to$E, 2, 4
6		$\to$I, 3–5

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Sub-proofs and Assumptions

1	$A \rightarrow C$		Pr.
2	$C \rightarrow B$		Pr.
WTS:	$A \rightarrow B$		
3		A	Ass.
4		C	$\rightarrow$E, 1, 3
5		B	$\rightarrow$E, 2, 4
6	$A \rightarrow B$		$\rightarrow$I, 3–5

However, there are some rules that can only be applied *if a sub-proof has occurred earlier in the proof*!

One such rule is $\rightarrow$I

"3–5" refers to the ***whole*** *sub-proof*!

Rule: Conditional Introduction ("→I" or "→In")

Conditional Introduction appeals to the following kind of reasoning:

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Since $\dfrac{\textbf{1.}\ \ P}{\therefore\ \ Q}$ is a valid argument, it must be true that $P \rightarrow Q$

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Since $\dfrac{\textbf{1.}\ \ P}{\therefore\ \ Q}$ is a valid argument, it must be true that $P \rightarrow Q$

If we **ASSUME** it’ll rain tonight, then it follows that I get my hair wet. So, ***no matter what*** *the weather’s like,* we know the conditional is true.

Rule: Conditional Introduction ("$\rightarrow$I" or "$\rightarrow$In")

Since $\begin{array}{l} 1.\ P \\ \hline \therefore\ Q \end{array}$ is a valid argument, it must be true that $P \rightarrow Q$

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Conditional Introduction, "$\rightarrow$I"

⋮	⋮	⋮
n	$\mathscr{A}$	Ass.
⋮	⋮	⋮
m	$\mathscr{B}$	(Some Rule) , (Line Numbers)
⋮	⋮	⋮
u	$\mathscr{A} \rightarrow \mathscr{B}$	$\rightarrow$I , n–m

Rule: Disjunction Elimination ("$\vee$E" or "$\vee$Out")

Disjunction Elimination *also* relies on the use of sub-proofs!

1. X is either in the B1 Cafeteria or in the 2nd-floor Cafe.

2. If X is in the B1 Cafeteria, then X is in 1555 Century Ave .

3. If X is in the 2nd-floor Cafe, then X is in 1555 Century Ave.

$\therefore$ X is in 1555 Century Ave!

Rule: Disjunction Elimination ("$\vee$E" or "$\vee$Out")

Disjunction Elimination *also* relies on the use of sub-proofs!

Rule: Disjunction Elimination ("∨E" or "∨Out")

Disjunction Elimination *also* relies on the use of sub-proofs!

Disjunction Elimination, "∨E"

i	$\mathscr{A} \vee \mathscr{B}$	(Some Rule) , (Line Numbers)
⋮	⋮	⋮
n	$\mathscr{A}$ (assumption, subproof)	Ass.
m	$\mathscr{C}$ (subproof)	(Some Rule) , (Line Numbers)
⋮		⋮
u	$\mathscr{B}$ (assumption, subproof)	Ass.
v	$\mathscr{C}$ (subproof)	(Some Rule) , (Line Numbers)
j	$\mathscr{C}$	∨E , i, n–m, u–v

Rule: Disjunction Elimination ("∨E" or "∨Out")

Disjunction Elimination *also* relies on the use of sub-proof**S**!

Disjunction Elimination, "∨E"

i	$\mathscr{A} \vee \mathscr{B}$	(Some Rule) , (Line Numbers)
⋮	⋮	⋮
n	│ $\mathscr{A}$	Ass.
m	│ $\mathscr{C}$	(Some Rule) , (Line Numbers)
⋮		⋮
u	│ $\mathscr{B}$	Ass.
v	│ $\mathscr{C}$	(Some Rule) , (Line Numbers)
j	$\mathscr{C}$	∨E , i, n–m, u–v

Rule: Negation Elimination ("¬E" or "𝕏In" or "⊥In")

Negation Elimination is also sometimes known as Contradiction Introduction, because it only "eliminates" negations in a special case, when its the negation of a sentence that appears elsewhere in your proof.

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Negation Elimination, “¬E”

⋮	⋮	⋮
n	$\mathscr{A}$	(Some Rule) , (Line Numbers)
⋮	⋮	⋮
m	$\neg\mathscr{A}$	(Some Rule) , (Line Numbers)
j	$\bot$	¬E , n, m

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Negation Elimination, "¬E"		
⋮	⋮	⋮
n	$\mathscr{A}$	(Some Rule) , (Line Numbers)
⋮	⋮	⋮
m	$\neg\mathscr{A}$	(Some Rule) , (Line Numbers)
j	$\mathbb{X}$	¬E , n, m

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⋮	⋮	⋮
n	$\mathscr{A}$	(Some Rule) , (Line Numbers)
⋮	⋮	⋮
m	$\neg\mathscr{A}$	(Some Rule) , (Line Numbers)
j	$\bot$	¬E , n, m

Rule: Negation Introduction ("¬I" or "¬In")

Negation introduction is a way of introducing a negation by "eliminating" a contradiction (when that contradiction arises **IN A SUB-PROOF!**). Specifically, if a sub-proof leads you to a contradiction, then you ***know*** that the assumptions you made *cannot* be true!

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Negation Introduction, "¬I"

⋮	⋮	⋮
n	$\mathscr{A}$	Ass.
⋮	⋮	⋮
m	$\bot$	(Some Rule) , (Line Numbers)
j	$\neg\mathscr{A}$	¬I , n–m

Rule: Indirect Proof ("IP")

Indirect Proof, also called "Proof by Contradiction", is extremely similar to negation introduction. However, instead of *introducing* a negation at the end, we're *assuming* a negated statement, in order to show that this leads to a contradiction. If assuming that "P" is false leads to a contradiction, then it must* be that P is true!

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Indirect Proof, "IP"

⋮	⋮	⋮
n	$\neg\mathscr{A}$	Ass.
⋮	⋮	⋮
m	$\bot$	(Some Rule) , (Line Numbers)
j	$\mathscr{A}$	IP , n–m

Rule: Explosion (“Ex” or “⊥E” or “$\mathbb{X}$Out”)

The Explosion rule has a fun name because it describes a really wild feature of TFL: Literally anything follows from a contradiction. That is, any argument with a contradiction among its premises is valid, no matter what else is in the argument!

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Explosion, "Ex"

n	$\bot$	(SOME RULE) , (LINE NUMBERS)
⋮	⋮	⋮
m	$\mathscr{A}$	Ex , n

How to Construct Good Proofs!

Work backwards from what you **want**

* More often than not, there are only a few rules that will be able to output a sentence like your desired conclusion.
* If you arrived at your concluding line using one of those rules, what other lines would the **rest** of your proof have to contain?

Work forwards from what you **have**

* If you've got some starting sentences, there are usually a few lines that you can do just by figuring out what you can extract from them.
* That is, use any elimination rules that are appropriate to the main connective of your premises, and "pull out" whatever you can into its own line.

How to Construct Good Proofs!

Give yourself mini-goals (but don't lose sleep if you miss 'em)

* When working backwards from your desired conclusion, make your mini-goal those sentences required to get to that point.
* When working forwards from your existing premises/lines, you might find a rule that you cannot use to work forward, because it requires a line you don't have. Make your mini-goal the lines that will allow you to use more of those rules.

Having trouble? Try Proof by Contradiction ("IP")

* If you're not sure of how to proceed, whether because you ran out of ways to work forward/back or because you're stumped, try this:
* Start a sub-proof, and assume **the negation of** your conclusion! Then, saying within that sub-proof, try to prove a contradiction ("$\perp$").

The rest of the Homework!

Chapter 15 Block C

Proofs 1, 2, 3, and 4.

1. $J \rightarrow \neg J \therefore \neg J$
2. $Q \rightarrow (Q \wedge \neg Q) \therefore \neg Q$
3. $A \rightarrow (B \rightarrow C) \therefore (A \wedge B) \rightarrow C$
4. $K \wedge L \therefore K \leftrightarrow L$

The rest of the Homework!

Chapter 15 Block C

. $J \rightarrow \neg J \therefore \neg J$

1	$J \rightarrow \neg J$	Pr.
2.	J	Ass.
3.	$\neg J$	$\rightarrow$E, 1, 2
4.	$\bot$	$\neg$E (aka $\bot$I), 2, 3
5.	$\neg J$	$\neg$I, 2–4

. $J \to \neg J \therefore \neg J$

The rest of the Homework!

Chapter 15 Block C

You can use the proof writer an chec er here:
https://proo s.openlo icpro ect.or

. $Q \rightarrow (Q \wedge \neg Q) \therefore \neg Q$

The rest of the Homework!

Chapter 15 Block C

The rest of the Homework!

Chapter 15 Block C

. $A \to (B \to C) \therefore (A \land B) \to C$

4. $K \wedge L \therefore K \leftrightarrow L$

Proof Practice!

Let's do some proofs together!

1. $\neg(P \wedge \neg P)$
2. $\neg\neg A \rightarrow A$
3. $\neg A \rightarrow (A \rightarrow \bot)$
4. $(A \vee \neg B) \rightarrow (B \rightarrow A)$

1.	$\neg A$	Ass.
2.	A	Ass.
3.	$\bot$	¬E / ⊥I, 1, 2
4.	$A \rightarrow \bot$	→I, 2—3
5.	$\neg A \rightarrow (A \rightarrow \bot)$	→I, 1—4

QUIZLET: Do ONE of these proofs along with me.
You can submit it here:
https://tinyurl.com/AttendQuizOct13

Proof Practice!

Let's do some proofs together!

$\neg(P \land \neg P)$

$\neg\neg A \rightarrow A$

$\neg A \rightarrow (A \rightarrow \bot)$

$(A \lor \neg B) \rightarrow (B \rightarrow A)$

You can use the proof writer an chec er here:
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Proof Practice!

Let's do some proofs together!

$\neg(P \wedge \neg P)$

$\neg\neg A \to A$

$\neg A \to (A \to \bot)$

$(A \vee \neg B) \to (B \to A)$

1.	$P \wedge \neg P$	Ass
2.	P	$\wedge$E, 1
3.	$\neg P$	$\wedge$E, 1
4.	$\bot$	$\neg$E, 2, 3
5.	$\neg(P \wedge \neg P)$	$\neg$I, 1–4

Proof Practice!

Let's do some proofs together!

$\neg(P \wedge \neg P)$

$\neg\neg A \rightarrow A$

$\neg A \rightarrow (A \rightarrow \bot)$

$(A \vee \neg B) \rightarrow (B \rightarrow A)$

1	$\neg\neg A$	Ass
2	$\neg A$	Ass
3	$\bot$	$\neg E$, 1, 2
4	A	IP, 2–3
5	$\neg\neg A \rightarrow A$	$\rightarrow I$, 1–4