Class 21 - More Natural Deduction in FOL!

Zee R. Perry

Let's do some logic!

Midterm starts Friday

The third midterm will start on Friday and be available until November 29th

Homework

This homework is due next **WEDNESDAY**, because it's a bit more substantial than our other previous ones. I'll also be working through some of the problems in class today, Monday, and next Wednesday, so you should tune in or watch the recordings to get help with the proofs.

***Re*-Read:** Chapters 34 to 36

Read: Chapters 37, 38 and 39

Do: The following problems in Chapter 34

- **Do:** Chapter 34, Block A, Correct both "proofs"
- **Do:** Chapter 34, Block B, Complete proof number 1
- **Do:** Chapter 34, Block E, Questions 1, 2, and 5

Do: The following problems in Chapter 37

- **Do:** Chapter 37, Block A, Questions 3 and 4

Chapter 34, Block E, Question 5

Chapter 34, Block E, Question 5

WTS: $\forall x R(x,x) \rightarrow \exists x \exists y R(x,y)$

Chapter 34, Block E, Question 5

WTS: $\forall x R(x,x) \rightarrow \exists x \exists y R(x,y)$

??	$\forall x R(x,x) \rightarrow \exists x \exists y R(x,y)$		→I, 1–??

Chapter 34, Block E, Question 5

WTS: $\forall x R(x,x) \rightarrow \exists x \exists y R(x,y)$

1	$\forall x R(x,x)$	Ass.
?	$\exists x \exists y R(x,y)$	∃I, ??
??	$\forall x R(x,x) \rightarrow \exists x \exists y R(x,y)$	→I, 1–??

Chapter 34, Block E, Question 5

WTS: $\forall x R(x,x) \rightarrow \exists x \exists y R(x,y)$

1	$\forall x R(x,x)$	Ass.
2	$R(b,b)$	$\forall$E, 1
?	$\exists x \exists y R(x,y)$	$\exists$I, ??
??	$\forall x R(x,x) \rightarrow \exists x \exists y R(x,y)$	$\rightarrow$I, 1–??

Chapter 34, Block E, Question 5

WTS: $\forall x R(x,x) \rightarrow \exists x \exists y R(x,y)$

1	$\forall x R(x,x)$	Ass.
2	$R(b,b)$	$\forall$E, 1
3	$\exists x R(x,x)$	$\exists$I, 2
?	$\exists x \exists y R(x,y)$	$\exists$I, ??
??	$\forall x R(x,x) \rightarrow \exists x \exists y R(x,y)$	$\rightarrow$I, 1–??

Chapter 34, Block E, Question 5

WTS: $\forall x R(x,x) \rightarrow \exists x \exists y R(x,y)$

1	$\forall x R(x,x)$	Ass.
2	$R(b,b)$	$\forall$E, 1
3	$\exists x R(x,x)$	$\exists$I, 2
4	$\exists x R(x,b)$	$\exists$I, 2
	$\exists y \exists x (Rxy)$	$\exists$I, y
?	$\exists x \exists y R(x,y)$	$\exists$I, ??
??	$\forall x R(x,x) \rightarrow \exists x \exists y R(x,y)$	$\rightarrow$I, 1–??

Chapter 34, Block E, Question 5

WTS: $\forall x R(x,x) \rightarrow \exists x \exists y R(x,y)$

1	$\forall x R(x,x)$	Ass.
2	$R(b,b)$	$\forall$E, 1
3	$\exists x R(x,x)$	$\exists$I, 2
4	$\exists x R(x,b)$	$\exists$I, 2
5	$\exists y R(b,y)$	$\exists$I, 2
?	$\exists x \exists y R(x,y)$	$\exists$I, ??
??	$\forall x R(x,x) \rightarrow \exists x \exists y R(x,y)$	$\rightarrow$I, 1–??

Chapter 34, Block E, Question 5

WTS: $\forall x R(x,x) \rightarrow \exists x \exists y R(x,y)$

1	$\forall x R(x,x)$	Ass.
2	$R(b,b)$	$\forall$E, 1
3	$\exists x R(x,x)$	$\exists$I, 2
4	$\exists x R(x,b)$	$\exists$I, 2
5	$\exists y R(b,y)$	$\exists$I, 2
6	$\exists x \exists y R(x,y)$	$\exists$I, 5
7	$\forall x R(x,x) \rightarrow \exists x \exists y R(x,y)$	$\rightarrow$I, 1–6

Rule: Existential Elimination ("∃E" or "∃Out")

Existential Elimination, "∃E"

Note: $\mathfrak{c}$ must **NOT** occur in any undischarged assumption before line n

Note-2: $\mathfrak{c}$ must **NOT** occur in "$\exists x \mathscr{A}(\ldots x \ldots x \ldots)$" !

Note-3: $\mathfrak{c}$ must **NOT** occur in "$\mathscr{B}$" !

n	$\exists x \mathscr{A}(\ldots x \ldots x \ldots)$	(Some Rule) , (Line Numbers)
m	$\mathscr{A}(\ldots \mathfrak{c} \ldots \mathfrak{c} \ldots)$	Ass.
⋮	⋮	⋮
u	$\mathscr{B}$	(Some Rule) , (Line Numbers)
v	$\mathscr{B}$	∃E , n, m–u

For fun: Chapter 34 Block C Question **Ferio**

Ferio: No G is an F. Some H is a G. Some H is not an F.

Quizlet:
tinyurl.com/AttendQuizNov17

Q1: Translate Ferio into FOL

Qbonus: *Work through this proof with me (by pausing the recording as you go)*

For fun: Chapter 34 Block C Question **Ferio**

1	$\forall x(G(x) \rightarrow \neg F(x))$	Pr.
2	$\exists x(H(x) \wedge G(x))$	Pr.

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2	$\exists x\,(H(x) \wedge G(x))$	Pr.
??	$\exists x\,(H(x) \wedge \neg F(x))$	∃E, 2, ??–??

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For fun: Chapter 34 Block C Question **Ferio**

1	$\forall x(G(x) \to \neg F(x))$	Pr.
2	$\exists x(H(x) \land G(x))$	Pr.
3	$H(a) \land G(a)$	Ass.
??	$\exists x(H(x) \land \neg F(x))$	$\exists$E, 2, ??–??

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2	$\exists x(H(x) \wedge G(x))$	Pr.
3	$H(a) \wedge G(a)$	Ass.
??	$\exists x(H(x) \wedge \neg F(x))$	∃I, ??
??	$\exists x(H(x) \wedge \neg F(x))$	∃E, 2, ??–??

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2	$\exists x(H(x) \land G(x))$	Pr.
3	$\quad H(a) \land G(a)$	Ass.
4	$\quad H(a)$	$\land$E, 3
??	$\quad \exists x(H(x) \land \neg F(x))$	$\exists$I, ??
??	$\exists x(H(x) \land \neg F(x))$	$\exists$E, 2, ??–??

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4	$\quad H(a)$	$\land$E, 3
5	$\quad G(a)$	$\land$E, 3
??	$\quad \exists x(H(x) \land \neg F(x))$	$\exists$I, ??
??	$\exists x(H(x) \land \neg F(x))$	$\exists$E, 2, ??–??

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3	$\quad H(a) \land G(a)$	Ass.
4	$\quad H(a)$	$\land$E, 3
5	$\quad G(a)$	$\land$E, 3
6	$\quad G(a) \to \neg F(a)$	$\forall$E, 1
??	$\quad \exists x(H(x) \land \neg F(x))$	$\exists$I, ??
??	$\exists x(H(x) \land \neg F(x))$	$\exists$E, 2, ??–??

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2	$\exists x(H(x) \wedge G(x))$	Pr.
3	$H(a) \wedge G(a)$	Ass.
4	$H(a)$	$\wedge$E, 3
5	$G(a)$	$\wedge$E, 3
6	$G(a) \rightarrow \neg F(a)$	$\forall$E, 1
7	$\neg F(a)$	$\rightarrow$E, 6,5
??	$\exists x(H(x) \wedge \neg F(x))$	$\exists$I, ??
??	$\exists x(H(x) \wedge \neg F(x))$	$\exists$E, 2, ??–??

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2	$\exists x(H(x) \wedge G(x))$	Pr.
3	$H(a) \wedge G(a)$	Ass.
4	$H(a)$	$\wedge$E, 3
5	$G(a)$	$\wedge$E, 3
6	$G(a) \rightarrow \neg F(a)$	$\forall$E, 1
7	$\neg F(a)$	$\rightarrow$E, 6,5
8	$H(a) \wedge \neg F(a)$	$\wedge$I, 4,7
??	$\exists x(H(x) \wedge \neg F(x))$	$\exists$I, ??
??	$\exists x(H(x) \wedge \neg F(x))$	$\exists$E, 2, ??–??

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2	$\exists x(H(x) \land G(x))$	Pr.
3	$H(a) \land G(a)$	Ass.
4	$H(a)$	$\land$E, 3
5	$G(a)$	$\land$E, 3
6	$G(a) \to \neg F(a)$	$\forall$E, 1
7	$\neg F(a)$	$\to$E, 6,5
8	$H(a) \land \neg F(a)$	$\land$I, 4,7
9	$\exists x(H(x) \land \neg F(x))$	$\exists$I, ??
??	$\exists x(H(x) \land \neg F(x))$	$\exists$E, 2, ??–??

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2	$\exists x(H(x) \land G(x))$	Pr.
3	$H(a) \land G(a)$	Ass.
4	$H(a)$	$\land$E, 3
5	$G(a)$	$\land$E, 3
6	$G(a) \to \neg F(a)$	$\forall$E, 1
7	$\neg F(a)$	$\to$E, 6,5
8	$H(a) \land \neg F(a)$	$\land$I, 4,7
9	$\exists x(H(x) \land \neg F(x))$	$\exists$I, 8
??	$\exists x(H(x) \land \neg F(x))$	$\exists$E, 2, ??–??

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3	$H(a) \land G(a)$	Ass.
4	$H(a)$	$\land$E, 3
5	$G(a)$	$\land$E, 3
6	$G(a) \to \neg F(a)$	$\forall$E, 1
7	$\neg F(a)$	$\to$E, 6,5
8	$H(a) \land \neg F(a)$	$\land$I, 4,7
9	$\exists x(H(x) \land \neg F(x))$	$\exists$I, 8
10	$\exists x(H(x) \land \neg F(x))$	$\exists$E, 2, ??–??

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2	$\exists x\,(H(x) \wedge G(x))$	Pr.
3	$H(a) \wedge G(a)$	Ass.
4	$H(a)$	$\wedge$E, 3
5	$G(a)$	$\wedge$E, 3
6	$G(a) \rightarrow \neg F(a)$	$\forall$E, 1
7	$\neg F(a)$	$\rightarrow$E, 6,5
8	$H(a) \wedge \neg F(a)$	$\wedge$I, 4,7
9	$\exists x\,(H(x) \wedge \neg F(x))$	$\exists$I, 8
10	$\exists x\,(H(x) \wedge \neg F(x))$	$\exists$E, 2, 3–9

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Q1: Translate Ferio into FOL

Qbonus: *Work through this proof with me (by pausing the recording as you go)*

Natural Deduction for FOL

Natural deduction for First-Order Logic is an extension of the natural deduction system for TFL.

That is:

Formatting: You keep the proof formatting rules, with vertical strokes marking proofs and sub-proofs, horizontal strokes after premises/assumption , numbered lines, etc.

Rules: You keep ALL the rules of TFL (both basic and defined)

Core **Connective** Rules of Natural Deduction for FOL

Connective rules that rely on previous lines in the *main* proof

Reit	Reiteration
$\wedge$**I**	Conjunction Introduction
$\wedge$**E**	Conjunction Elimination
$\vee$**I**	Disjunction Introduction
$\rightarrow$**E**	Conditional Elimination
$\leftrightarrow$**E**	Biconditional Elimination

Connective rules that *also* (generally) occur *within* or care *about* sub-proofs

$\rightarrow$**I**	Conditional Introduction
$\leftrightarrow$**I**	Biconditional Introduction
$\vee$**E**	Disjunction Elimination
$\neg$**E**	Negation Elimination
$\neg$**I**	Negation Introduction
IP	Indirect Proof
Ex	Explosion

(These are *all* the same as they are in TFL proof theory.)

Core **Quantifier** Rules of Natural Deduction for FOL

Quantifier rules that only rely on previous *lines*

$\forall$E Universal Elimination

$\exists$I Existential Introduction

Quantifier rules that either occur *within* or care *about sub-proofs*

$\forall$I Universal Introduction

$\exists$E Existential Elimination

Rule: Universal Elimination (“∀E” or “∀Out”)

1. Everything is on sale for 50% off!.

∴ This apple is on sale for 50% off!.

Rule: Universal Elimination ("∀E" or "∀Out")

1. Everything is on sale for 50% off!.

∴ This apple is on sale for 50% off!.

1. $\forall x\big(F(x) \wedge R(x,m)\big)$

$\therefore\ F(a) \wedge R(a,m)$

Rule: Universal Elimination ("∀E" or "∀Out")

1. Everything is on sale for 50% off!.

$\therefore$ This apple is on sale for 50% off!.

1. $\forall x\big(F(x) \wedge R(x, m)\big)$

$\therefore$ $F(a) \wedge R(a, m)$

1. $\forall x\big(D(x) \rightarrow G(x)\big)$

$\therefore$ $D(e) \rightarrow G(e)$

Rule: Universal Elimination ("∀E" or "∀Out")

1. Everything is on sale for 50% off!.

∴ This apple is on sale for 50% off!.

1. $\forall x\big(F(x) \wedge R(x,m)\big)$

∴ $F(b) \wedge R(b,m)$

1. $\forall x\big(D(x) \rightarrow G(x)\big)$

∴ $D(e) \rightarrow G(e)$

Universal Elimination, "∀E"

n	$\forall x \mathscr{R}(\ldots x \ldots x \ldots)$	(Some Rule) , (Line Numbers)	
⋮	⋮	⋮	
u	$\mathscr{R}(\ldots \mathfrak{c} \ldots \mathfrak{c} \ldots)$	∀E ,	n

Rule: Existential Introduction ("∃I" or "∃Intro")

1. Sophie is too tired to watch a movie at Stella's house.

∴ Someone is too tired to watch a movie at Stella's house.

Rule: Existential Introduction ("∃I" or "∃Intro")

1. Sophie is too tired to watch a movie at Stella's house.

∴ Someone is too tired to watch a movie at Stella's house.

1. $F(a) \wedge R(a, m)$

∴ $\exists x\big(F(x) \wedge R(x, m)\big)$

Rule: Existential Introduction ("∃I" or "∃Intro")

1. Sophie is too tired to watch a movie at Stella's house.

∴ Someone is too tired to watch a movie at Stella's house.

1. $F(a) \wedge R(a,m)$

$\therefore\ \exists x\big(F(x) \wedge R(x,m)\big)$

1. $\exists x\big(D(e) \vee G(x,e)\big)$

$\therefore\ \exists y \exists x\big(D(y) \vee G(x,e)$

Rule: Existential Introduction (“∃I” or “∃Intro”)

1. Sophie is too tired to watch a movie at Stella’s house.

$\therefore$ Someone is too tired to watch a movie at Stella’s house.

1. $F(a) \wedge R(a, m)$

$\therefore$ $\exists x\big(F(x) \wedge R(x, m)\big)$

1. $\exists x\big(D(e) \vee G(x, e)\big)$

$\therefore$ $\exists y \exists x\big(D(y) \vee G(x, e$

Existential Introduction, “∃I”

n	$\mathscr{R}(\ldots\mathfrak{c}\ldots\mathfrak{c}\ldots)$	(Some Rule) , (Line Numbers)	
⋮	⋮	⋮	
u	$\exists x \mathscr{R}(\ldots x \ldots \mathfrak{c} \ldots)$	∃I ,	n

(note: x must **NOT** occur anywhere in “$\mathscr{R}(\ldots\mathfrak{c}\ldots\mathfrak{c}\ldots)$” !!)

Rule: Universal Introduction ("∀I" or "∀Intro")

This is a rule that's easier to explain in the context of a proof than in a natural language context.

Here's the idea:

1

Rule: Universal Introduction ("∀I" or "∀Intro")

This is a rule that's easier to explain in the context of a proof than in a natural language context.

Here's the idea:

1	$F(a) \wedge G(a)$	Ass.

Rule: Universal Introduction ("∀I" or "∀Intro")

This is a rule that's easier to explain in the context of a proof than in a natural language context.

Here's the idea:

1	$F(a) \wedge G(a)$	Ass.
2	$G(a)$	$\wedge$ E, 1

Rule: Universal Introduction ("∀I" or "∀Intro")

This is a rule that's easier to explain in the context of a proof than in a natural language context.

Here's the idea:

1	$F(a) \wedge G(a)$	Ass.
2	$G(a)$	$\wedge$ E, 1
3	$F(a)$	$\wedge$ E, 1
NO	$\forall x Fx$	∀I, 3

Rule: Universal Introduction ("∀I" or "∀Intro")

This is a rule that's easier to explain in the context of a proof than in a natural language context.

Here's the idea:

1	$F(a) \wedge G(a)$	Ass.
2	$G(a)$	$\wedge$ E, 1
3	$F(a)$	$\wedge$ E, 1
4	$G(a) \wedge F(a)$	$\wedge$ I, 2, 3

Rule: Universal Introduction ("∀I" or "∀Intro")

This is a rule that's easier to explain in the context of a proof than in a natural language context.

Here's the idea:

1	$F(a) \wedge G(a)$	Ass.
2	$G(a)$	$\wedge$ E, 1
3	$F(a)$	$\wedge$ E, 1
4	$G(a) \wedge F(a)$	$\wedge$ I, 2, 3
5	$\big(F(a) \wedge G(a)\big) \rightarrow \big(G(a) \wedge F(a)\big)$	$\rightarrow$I, 1–4

Rule: Universal Introduction ("∀I" or "∀Intro")

This is a rule that's easier to explain in the context of a proof than in a natural language context.

Here's the idea:

1	$F(a) \wedge G(a)$	Ass.
2	$G(a)$	$\wedge$ E, 1
3	$F(a)$	$\wedge$ E, 1
4	$G(a) \wedge F(a)$	$\wedge$ I, 2, 3
5	$\big(F(a) \wedge G(a)\big) \rightarrow \big(G(a) \wedge F(a)\big)$	$\rightarrow$I, 1–4
6	$\forall x\big([F(x) \wedge G(x)] \rightarrow [G(x) \wedge F(x)]\big)$	$\forall$I, 5

Concept: Discharging Assumptions

When is an assumption **Undischarged**?

When it's a premise of your proof, or when it's the assumption in a sub-proof that hasn't been closed (i.e. hasn't *ended*).

How can an assumption be **Discharged**?

When it's the assumption in a sub-proof, then you have to close/end the sub-proof.
(i.e. have your current line be *outside* the sub-proof).

The takeaway: There is no way to use $\forall I$ on a sentence with a name that appears in your premises (if you're anywhere in the proof) or in any of your assumptions (if within a sub-proof).

Rule: Universal Introduction (“∀I” or “∀Intro”)

Universal Introduction, “∀I”

Note: $\mathfrak{c}$ must **NOT** occur in any undischarged assumption!

n	$\mathscr{A}(\ldots\mathfrak{c}\ldots\mathfrak{c}\ldots)$	(Some Rule) , (Line Numbers)
⋮	⋮	⋮
u	$\forall x \mathscr{A}(\ldots x \ldots x \ldots)$	∀I , n

x must not occur in “$\mathscr{A}(\ldots\mathfrak{c}\ldots\mathfrak{c}\ldots)$” !

Rule: Existential Elimination ("∃E" or "∃Out")

This is another rule that's easier to explain in the context of a proof:

Here's the idea: You make up an object (or make up a NAME for one), without assuming anything else about it besides whatever the existential asserts.

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1	$\exists x \big(F(x) \wedge G(x)\big)$	Pr.

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1	$\exists x\big(F(x) \wedge G(x)\big)$	Pr.
WTS:	$\exists x\big(F(x)\big)$	

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1	$\exists x\big(F(x) \wedge G(x)\big)$	Pr.
WTS:	$\exists x\big(F(x)\big)$	
2	$F(a) \wedge G(a)$	Ass.

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1	$\exists x\big(F(x) \wedge G(x)\big)$	Pr.
WTS:	$\exists x\big(F(x)\big)$	
2	$F(a) \wedge G(a)$	Ass.
3	$F(a)$	$\wedge$E, 2

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WTS:	$\exists x\big(F(x)\big)$	
2	$F(a) \wedge G(a)$	Ass.
3	$F(a)$	$\wedge$E, 2
4	$\exists x F(x)$	∃I, 3

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1	$\exists x\big(F(x) \wedge G(x)\big)$	Pr.
WTS:	$\exists x\big(F(x)\big)$	
2	$\quad F(a) \wedge G(a)$	Ass.
3	$\quad F(a)$	∧E, 2
4	$\quad \exists x F(x)$	∃I, 3
5	$\exists x F(x)$	∃E, 1–4

Rule: Existential Elimination (“∃E” or “∃Out”)

Existential Elimination, “∃E”

Note: $\mathfrak{c}$ must **NOT** occur in any undischarged assumption before line *n*

Note-2: $\mathfrak{c}$ must **NOT** occur in “$\exists x \mathscr{A}(\ldots x \ldots x \ldots)$” !

Note-3: $\mathfrak{c}$ must **NOT** occur in “$\mathscr{B}$” !

n	$\exists x \mathscr{A}(\ldots x \ldots x \ldots)$	(Some Rule) , (Line Numbers)
m	$\quad \mathscr{A}(\ldots \mathfrak{c} \ldots \mathfrak{c} \ldots)$	Ass.
⋮	$\quad \vdots$	⋮
u	$\quad \mathscr{B}$	(Some Rule) , (Line Numbers)
v	$\mathscr{B}$	∃E , n, m–u

Core **Quantifier** Rules of Natural Deduction for FOL

Quantifier rules that only rely on previous *lines*

∀E	Universal Elimination
∃I	Existential Introduction

Quantifier rules that either occur *within* or care *about sub-proofs*

∀I	Universal Introduction
∃E	Existential Elimination

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CQ	Conversion of Quantifiers		

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∃I	Existential Introduction	∃E	Existential Elimination
CQ	Conversion of Quantifiers		
=I	Identity Introduction		
=E	Identity Elimination		

Think about what Quantifiers mean

What does it mean when we say "$\neg\exists x\big(U(x)\big)$"?

Suppose '$U(x)$' = 'x is a unicorn'

$\neg$ $\exists x$ $U(x)$

Literal: "It's not the case there exists an x that is a unicorn"

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Fool: I wonder if there exist any unicorns? I might've seen one!

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You: No you didn't. There aren't any unicorns. So whatever you saw wasn't a unicorn, because ***everything*** isn't a unicorn!

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Moral of the story: Another way we can say that there are no unicorns is to say that *everything* is *not* a unicorn.

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in FOL: "$\forall x\big(\neg U(x)\big)$"

Changing Quantifiers by moving Negations

So that means that saying "$\neg\exists x\Big(U(x)\Big)$" ("There does not exist anything that is a unicorn") is equivalent to saying "$\forall x\Big(\neg U(x)\Big)$" ("Everything is not a unicorn").

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What would it mean to say that that "Not everyone is a vegetarian"? Let's say the domain = all people, and '$V(x)$'='x is a vegetarian':

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Translation: "$\neg\forall x\Big(V(x)\Big)$"

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Let's say the domain = all people, and '$V(x)$'='x is a vegetarian':

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How would we express the same sentiment using the Existential quantifier?

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Translation: "$\exists x\big(\neg V(x)\big)$"

Fun Fact

The universal quantifier, ‘$\forall$’, and the existential quantifier, ‘$\exists$’, are **Logical Duals.**

When two concepts are logical duals, we can translate between them in symmetric ways by moving or adding negations.

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Other examples

Permitted/Required: "You are *not* **permitted** to park there" $\leftrightarrow$ "You are **required** to *not* park there"

Necessary/Possible: "It's *not* **possible** to go faster than light" $\leftrightarrow$ "It's necessarily **true** that you *don't* go faster than light"

Rule: Conversion of Quantifiers ("CQ" or "$\exists/\forall$")

The conversion of quantifiers rule lets you swap one quantifier for another by moving the negation accordingly.

1. There do not exist any unicorns

$\therefore$ Everything is not a unicorn

Rule: Conversion of Quantifiers ("CQ" or "$\exists/\forall$")

The conversion of quantifiers rule lets you swap one quantifier for another by moving the negation accordingly.

1. Not everyone is a vegetarian

$\therefore$ There exists at least one non-vegetarian

Rule: Conversion of Quantifiers (“CQ” or “∃/∀”)

The conversion of quantifiers rule lets you swap one quantifier for another by moving the negation accordingly.

1. Not everyone is a vegetarian

$\therefore$ There exists at least one non-vegetarian

1. $\forall x \neg \big(F(x) \wedge R(x, m) \big)$

$\therefore \quad \neg E x \big(F(x) \wedge R(x, m) \big)$

Rule: Conversion of Quantifiers ("CQ" or "∃/∀")

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1. Not everyone is a vegetarian

∴ There exists at least one non-vegetarian

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∴ $\neg Ex\big(F(x) \wedge R(x, m)\big)$

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∴ $\neg Ex \big(F(x) \wedge R(x,m)\big)$

1. $\exists x \neg \big(D(x) \vee G(x)\big)$

∴ $\neg \forall x \big(D(x) \vee G(x)\big)$

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The conversion of quantifiers rule lets you swap one quantifier for another by moving the negation accordingly.

1. Not everyone is a vegetarian

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$$\begin{array}{ll} \mathbf{1.} & \forall x \neg\big(F(x) \wedge R(x,m)\big) \\ \hline \therefore & \neg Ex\big(F(x) \wedge R(x,m)\big) \end{array}$$

$$\begin{array}{ll} \mathbf{1.} & \exists x \neg\big(D(x) \vee G(x)\big) \\ \hline \therefore & \neg \forall x\big(D(x) \vee G(x)\big) \end{array}$$

Conversion of Quantifiers, "CQ" (all four ways)					
n	$\neg\exists\mathscr{A}$		n	$\exists\neg\mathscr{A}$	
m	$\forall\neg\mathscr{A}$	CQ , n	m	$\neg\forall\mathscr{A}$	CQ , n
n	$\neg\forall\mathscr{A}$		n	$\forall\neg\mathscr{A}$	
m	$\exists\neg\mathscr{A}$	CQ , n	m	$\neg\exists\mathscr{A}$	CQ , n

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Quantifier rules that only rely on previous *lines*

$\forall$E Universal Elimination

$\exists$I Existential Introduction

CQ Conversion of Quantifiers

$=$I Identity Introduction

$=$E Identity Elimination

Quantifier rules that either occur *within* or care *about sub-proofs*

$\forall$I Universal Introduction

$\exists$E Existential Elimination

Rule: Identity Introduction ("=I" or "ID")

c = c is a tautology

This is the simplest rule you'll ever meet. In fact, it's a rule that doesn't require *any* lines at all!

Identity Introduction "=I"		
n	$\mathfrak{c} = \mathfrak{c}$	=I

1	$d = d$	=I

Rule: Identity Elimination ("=E" or "LL")

Identity elimination is a bit more substantive, but it's still pretty easy to get.

$a = b$ means 'a' and 'b' refer to the same object

1. Superman = Clark Kent.
2. Superman can fly.

∴ Clark Kent can fly.

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1. Superman = Clark Kent.
2. Clark Kent lives in Clark Kent's house.

∴ Superman lives in Clark Kent's house.

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Identity elimination is a bit more substantive, but it's still pretty easy to get.

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Identity Elimination "=E"

n	$\mathfrak{a} = \mathfrak{b}$	(Some Rule) , (Line Numbers)
m	$\mathscr{R}(\ldots\mathfrak{a}\ldots\mathfrak{a}\ldots)$	(Some Rule) , (Line Numbers)
⋮	⋮	⋮
u	$\mathscr{R}(\ldots\mathfrak{b}\ldots\mathfrak{a}\ldots)$	=E , n,m

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CQ Conversion of Quantifiers

=I Identity Introduction

=E Identity Elimination

Quantifier rules that either occur *within* or care *about sub-proofs*

∀I Universal Introduction

∃E Existential Elimination

Homework

This homework is due next **WEDNESDAY**, because it's a bit more substantial than our other previous ones. I'll also be working through some of the problems in class today, Monday, and next Wednesday, so you should tune in or watch the recordings to get help with the proofs.

***Re*-Read:** Chapters 34 to 36

Read: Chapters 37, 38 and 39

Do: The following problems in Chapter 34

- **Do:** Chapter 34, Block A, Correct both "proofs"
- **Do:** Chapter 34, Block B, Complete proof number 1
- **Do:** Chapter 34, Block E, Questions 1, 2, and ~~5~~

Do: The following problems in Chapter 37

- **Do:** Chapter 37, Block A, Questions 3 and 4